HW4, for MATH441, STAT461, STAT561, due October 2nd

1. Consider rolling a 6-sided die and a 4-sided die. Let X be the value of the 6-sided die minus the value of the 4-sided die.

(a) Determine the probability mass function for X.

(b) Determine $E[X]$.

(c) Suppose with probability p I pick the 6-sided die and with probability $1-p$ I pick the 4-sided die. Then I roll whichever die I picked up. Let Y be the resulting value. Find $E[Y]$. Hint: First find the mass function for Y.

2. Let X be the value of a single fair die with 4 sides.

(a) Find $\sqrt{E[X]}$.

(b) Find $E[\sqrt{X}]$.

(c) Find the standard deviation of X, $\sqrt{Var(X)} = \sqrt{E\{(X-E[X])^2\}}$

(d) Find $E(|X-E[X]|) = E\{\sqrt{(X-E[X])^2}\}$.

3. Suppose the number of accidents per day at a certain intersection is a Poisson random variable with rate 0.1 on days that it is not raining but with rate 0.2 on days when there is rain.

(a) Find the probability that there is more than one accident on a rainy day.

(b) Find the probability that there is more than one accident on a non-rainy day.

(c) If the probability that there is rain tomorrow is 80%, find the probability that there will be more than one accident.

4. Let

$$P(X=i) = \begin{cases} e^{-3}\cdot 3^i/i! & \text{for } i \in \{0,1,2,\dots\} \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $P(X \le 2)$

(b) Show that $\sum_{i=0}^{\infty} P(X=i) = 1$. Hint: Recall the infinite series for e^x or look up and read about the Poisson distribution either in one of the textbooks or online.

(c) (Graduate students). Find $P(X$ is an odd number$)$.

For part (c), write the probability as an infinite series, and see if you find what infinite series this corresponds to from Calc II or Wikipedia.

5. When I play drums, the number of times I drop a stick is a Poisson random variable with a rate of once every 20 minutes. What is the probability that I drop the stick exactly two times in a 45-minute practice session?

6. Suppose I flip a fair coin $2n$ times. Find the probability that I get exactly n heads when

(a) $n=2$,
(b) $n=4$,
(c) $n=8$,
(d) $n=9$,
(e) $n=100$,

Now find the probability that, from the same fair coin, I get at least 60% of the flips are heads for the same values of n.

7. Suppose you roll a (fair, six-sided) die once and record the value.

(a) Now roll the die again until something other than the first value shows up. Let X_2 be this waiting time (the number of new rolls until the second distinct value is observed). What is the distribution for X_2 (it is one of the named distribution in chapter 4 of Ross), and what is $E[X_2]$?

(b) Now that two distinct values have been rolled, keep rolling again until a third distinct value has been rolled. Let this waiting time be $E[X_3]$. What is the distribution of X_3 and what is $E[X_3]$?

(c) (graduate students). Define X_i as the waiting time until the ith distinct value is observed after $i-1$ distinct values have been observed. How do you interpret the sum $\sum_{i=1}^{6} X_i$ and what is the value of $E[\sum_{i=1}^{6} X_i]$? Generalize this problem to an m-sided fair die.