Homework 9

1. Let X and Y be independent and identically distributed where X has density

$$f_X(x) = \frac{1}{x^2}\, I(x > 1)$$

Let $U = X/Y$, $V = X$. Find the joint density for (U, V). Also find the marginal density $f_U(u)$.

2. Let X and Y be independent and identically distributed exponential random variables with rate λ. Let $U = X/Y$ and let $V = XY$. Find the joint desnity for (U, V). Also find the marginal densities $f_U(u)$ and $f_V(v)$.

3. Let U_1 and U_2 be uniform(0,b), i.e., they have density

$$f(u) = \frac{1}{b}\ \ I(0 < u < b)$$

Find the density for $U = U_1 + U_2$. You can use a convolution to solve this or a bivariate transformation, or just by using the CDF method.

4. Let U be uniform(0,1).

(a) Find the density for $X = 1 - U$.

(b) Find the density for $Y = 2 - U$.

5. Considering rolling two 8-sided dice, where the two dice are independent. Let X be the value of the first die and Y the value of the sum of the two dice. Find the joint moment generating function of X and Y.