1. Let $X$ and $Y$ be independent and identically distributed where $X$ has density

$$f_X(x) = \frac{1}{x^2} I(x > 1)$$

Let $U = X/Y$, $V = X$. Find the joint density for $(U, V)$. Also find the marginal density $f_U(u)$.

2. Let $X$ and $Y$ be independent and identically distributed exponential random variables with rate $\lambda$. Let $U = X/Y$ and let $V = XY$. Find the joint density for $(U, V)$. Also find the marginal densities $f_U(u)$ and $f_V(v)$.

3. Let $U_1$ and $U_2$ be uniform$(0, b)$, i.e., they have density

$$f(u) = \frac{1}{b} I(0 < u < b)$$

Find the density for $U = U_1 + U_2$. You can use a convolution to solve this or a bivariate transformation, or just by using the CDF method.

4. Let $U$ be uniform$(0,1)$.

(a) Find the density for $X = 1 - U$.

(b) Find the density for $Y = 2 - U$.

5. Considering rolling two 8-sided dice, where the two dice are independent. Let $X$ be the value of the first die and $Y$ the value of the sum of the two dice. Find the joint moment generating function of $X$ and $Y$. 