Last Homework, Due Friday Dec 8th

1. Let X and Y be independent and identically distributed where X has density

$$f_X(x) = \frac{1}{x^2}\ I(x > 1)$$

Let $U = X/Y$, $V = X$. Find the joint density for (U, V). Also find the marginal density $f_U(u)$.

2. Let X and Y be independent and identically distributed exponential random variables with rate λ. Let $U = X/Y$ and let $V = XY$. Find the joint desnity for (U, V). Also find the marginal densities $f_U(u)$ and $f_V(v)$.

3. Let U_1 and U_2 be uniform(0,b), i.e., they have density

$$f(u) = \frac{1}{b}\ \ I(0 < u < b)$$

Find the density for $U = U_1 + U_2$. You can use a convolution to solve this or a bivariate transformation, or just by using the CDF method.

4. Let U be uniform$(0, 1)$, and let V be uniform$(0, U)$.

(a) Find $E[V|U = u]$.

(b) Find $Var(V|U = u)$.

(c) Find $E[U]$

(d) Find $Var(U)$.

Parts (e)–(f) are for graduate students:

(e) Find the joint distriubtion $f_{U,V}(u, v)$. Hint: You can use the fact that $f_{U,V}(u, v) = f_{V|U}(v|u)f_U(u)$. Be sure to specify appropriate limits for u and v.

(f) Find the marginal distribution for $f_V(v)$, specifying appropriate limits for v.

5. Considering rolling two 8-sided dice, where the two dice are independent. Let X be the value of the first die and Y the value of the sum of the two dice. Find the joint moment generating function of X and Y.