Test 1, ENCI303, 23 March 2010

Formula Sheet

$$P(A) = P(A \,|\, B_1)P(B_1) + \cdots + P(A \,|\, B_n)P(B_n)$$

$$P(B_i \,|\, A) = \frac{P(A \,|\, B_i)P(B_i)}{\sum_i P(A \,|\, B_i)P(B_i)}$$

$$P(A \cup B) = P(A) + P(B) - P(AB)$$

$$Var(X) = E(X^2) - (E(X))^2$$

Binomial distribution: $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, $\binom{n}{x} = \frac{n!}{x!(n-x)!}$

Poisson distribution with rate λ: $P(X = x) = e^{-\lambda}\lambda^x/x!$

INSTRUCTIONS:

1. For two events A and B, if $p = P(A \,|\, B)$, which of the following is always equal to $1 - p$:

(a) $P(A^c|B^c)$
(b) $P(A^c|B)$
(c) $P(A|B^c)$
(d) $P(B|A)$

2. Suppose two events, A and B, are dependent, with $0 < P(A) < 1$, $0 < P(B) < 1$. Which of the following can be said about A and B^c?

(a) A and B^c are independent.
(b) A and B^c are mutually exclusive.
(c) A and B^c are dependent.
(d) none of the above.

3. Suppose I flip a fair coin. If I get Heads, then I roll a 6-sided die. If I get Tails, then I roll two 6-sided dice and count the total.

(a) Let X be the value of the die or dice that I get. Find $P(X = 6)$.

(b) Suppose that I get $X = 6$. Given that I got $X = 6$, what is the probability that the coin landed Heads?

4. Suppose the travel time between two major cities A and B by air is 6 or 7 hr if the flight is nonstop; however, if there is one stop, the travel time would be 9, 10, or 11 hr. A nonstop flight between A and B would cost $1200, whereas with one stop the cost is only $550. Then, between cities B and C, all flights are nonstop requiring 2 or 3 hours at a cost of $300. For a passenger wishing to travel from city A to city C,

(a) What is the possibility space or sample space of his travel times from A to B? From A to C?

(b) What is the sample space of his travel cost from A to C?

(c) If T = travel time from city A to city C, and S = cost of travel from A to C, what is the sample space of T and S?

5. The maximum intensity of the next earthquake in a city may be classified (for simplicity) as low (L), medium (M), or high (H) with relative likelihoods of 15:4:1. Suppose also that buildings may be divided into two types; poorly constructed (P) and well constructed (W). About 20% of all the buildings in the city are known to be poorly constructed for earthquake resistance.

It is estimated that a poorly constructed building will be damaged with a probability of 0.10, 0.50, or 0.90 when subjected to a low-, medium-, or high-intensity earthquake, respectively. However, a well-constructed building will survive a low-intensity earthquake, although it may be damaged when subjected to a medium- or high-intensity earthquake with probability of 0.05 or 0.20, respectively.

(a) What is the probability that a well-constructed building will be damaged during the next earthquake?

(b) What proportion of the buildings in this city will be damaged during the next earthquake?

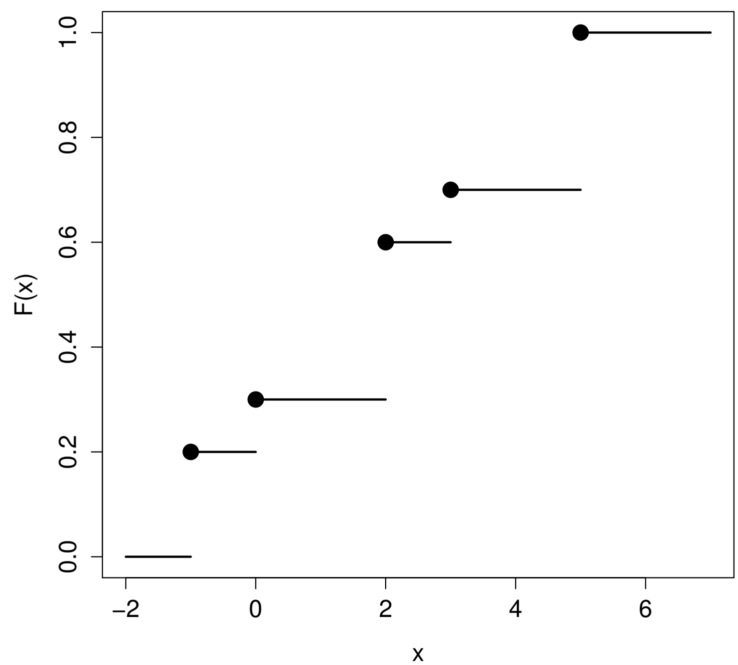
(c) If a building in the city is damaged after an earthquake, what is the probability that the building was poorly constructed?

6. The number of traffic accidents at a particular intersection has a Poisson with a rate of 1 in every 3 yr.

(a) What is the probability that there will not be any accidents in the next five years?

(b) Suppose that every accident at this intersection has a 5% chance of fatality. What is the probability that there is a fatal accident within the next 3 years?

7. Consider the cdf on the following page, which is only plotted from $x = -2$ to $x = 7$:



Answer the following questions:

(a) What is $P(X = -1)$?

(b) What is $P(X \leq 1)$?

(c) What is $P(X < 1)$

(d) What is $P(0 < X < 2)$?

(e) Write the probability mass function $P(X = i)$. Be sure to specify the mass function for all real values i.