MATH 180–Elements of Calculus
Exam 1
Dr. Janet Vassilev

(1) Evaluate and simplify $\dfrac{32^{\frac{2}{5}}32^{\frac{3}{10}}}{32^{\frac{7}{10}}}$.

$$\frac{32^{2/5}32^{3/10}}{32^{7/10}} = 32^{2/5+3/10-7/10} = 32^{4/10+3/10-7/10} = 32^0 = \boxed{1}$$

(2) Rationalize the denominator of $\dfrac{3}{1+\sqrt{2}}$.

$$\frac{3}{1+\sqrt{2}} = \frac{3}{(1+\sqrt{2})}\frac{(1-\sqrt{2})}{(1-\sqrt{2})} = \frac{3-3\sqrt{2}}{1-2} = \frac{3-3\sqrt{2}}{-1} = \boxed{3\sqrt{2}-3}$$

(3) Find the equation of the line which passes through the point $(10, -3)$ and is parallel to the line $3x - 5y + 15 = 0$ in slope-intercept form.

Solve for y: $5y = 3x+15$ or $y = \underline{\underline{3/5}}\,x+3$

The slope of the parallel line is $3/5$.

So $y - -3 = 3/5(x-10)$ or $y = 3/5\,x - 6 - 3$

or $\boxed{y = 3/5\,x - 9}$

(4) Let $f(x) = \begin{cases} \dfrac{2+x}{3-x} & \text{if } x \le 1 \\ 1+2x^3 & \text{if } 1 < x < 3 \\ 4 & \text{if } x \ge 3 \end{cases}$

Find $f(-1)$, $f(2)$ and $f(3)$.

$$f(-1) = \frac{2+-1}{3-(-1)} = \boxed{\frac{1}{4}}$$

$$f(2) = 1+2\cdot 2^3 = \boxed{17}$$

$$f(3) = \boxed{4}$$

(5) Find the domain of $f(x) = \sqrt{x^2 - 5x - 6}$.

$x^2 - 5x - 6 \geq 0$ or $(x-6)(x+1) \geq 0$

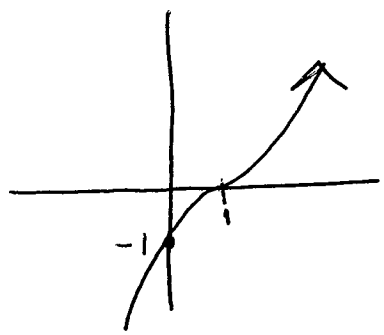
so $(x-6) \geq 0$ and $(x+1) \geq 0 \Rightarrow x \geq 6$

or $x - 6 \leq 0$ and $x+1 \leq 0 \Rightarrow x \leq -1$

Thus $x \geq 6$ or $x \leq -1$

So $\boxed{D = \{x \mid x \geq 6 \text{ or } x \leq -1\} \text{ or } (-\infty, 1] \cup [6, \infty)}$

(6) Sketch the graph of $f(x) = (x-1)^3$.



(7) Factor $a^3b - 4a^2b^2 - 21ab^3$.

$a^3b - 4a^2b^2 - 21ab^3 = ab(a^2 - 4ab - 21b^2)$

$= \boxed{ab(a-7b)(a+3b)}$

(8) Let $f(x) = x^2 + 4x$ and $g(x) = \sqrt{x+4}$.

(a) What is $\dfrac{f(x)}{g(x)}$ in simplified form?

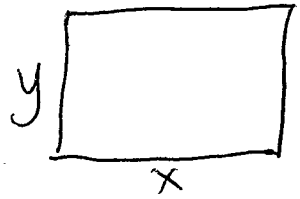
$\dfrac{f(x)}{g(x)} = \dfrac{x^2+4x}{\sqrt{x+4}} = \dfrac{x(x+4)}{\sqrt{x+4}} = x\sqrt{x+4}$

for $x > -4$

(b) What is $g(f(x))$ in simplified form?

$g(f(x)) = \sqrt{x^2+4x+4} = \sqrt{(x+2)^2} = |x+2|$

(9) John plans to make a rectangular enclosure for his horses with 200 yards of fencing. If x is the width of the enclosure, find a function for the area of the enclosure in terms of x.



$2x+2y=200 \Rightarrow y=100-x$

$A = xy = x(100-x)$

or $\boxed{A(x) = x(100-x)}$

(10) Find the limits of the following if they exist:

(a) $\lim_{x\to -2} \frac{x^2+2x}{x^2-4} = \lim_{x\to -2} \frac{(x+2)x}{(x+2)(x-2)} = \lim_{x\to -2} \frac{x}{x-2} = \frac{-2}{-2-2} = \boxed{\frac{1}{2}}$

(b) If $f(x) = \begin{cases} 3-x \text{ if } x<2 \\ \sqrt{x-1} \text{ if } x \geq 2, \end{cases}$ then $\lim_{x\to 2} f(x) = \lim_{x\to 2^+} \sqrt{x-1} = \sqrt{2-1} = 1$

$= \lim_{x\to 2^-} 3-x = 3-2 = \boxed{1}$

Since the left & right limits are the same.

(c) $\lim_{x\to -3} \frac{5+x}{6-2x} = \boxed{\text{Does not exist}}$

Since the top approaches 8 but the bottom gets small (close to 0)

(d) $\lim_{x\to\infty} \frac{x^2+1}{3-x} = \lim_{x\to\infty} \frac{(x^2+1)\frac{1}{x}}{(3-x)\frac{1}{x}} = \lim_{x\to\infty} \frac{x+\frac{1}{x}}{\frac{3}{x}-1}$

$\frac{1}{x} \to 0$ so $\lim_{x\to\infty} \frac{x^2+1}{3-x} = \lim_{x\to\infty} \frac{x}{-1} = \boxed{-\infty}$

(e) $\lim_{x\to -\infty} \frac{7x+1}{6+5x} = \lim_{x\to -\infty} \frac{(7x+1)(\frac{1}{x})}{(6+5x)(\frac{1}{x})} = \lim_{x\to -\infty} \frac{7+\frac{1}{x}}{\frac{6}{x}+5} = \boxed{\frac{7}{5}}$

(11) If $\lim_{x\to 4} f(x) = 7$ and $\lim_{x\to 4} g(x) = -6$, find the following:

(a) $\lim_{x\to 4} f(x) - g(x) =$ $7-(-6)=13$

(b) $\lim_{x\to 4} f(x)g(x) =$ $7(-6)=-42$

(12) Let $f(x) = \begin{cases} 10 \text{ if } x < -5 \\ -2x \text{ if } -5 \le x < -2 \\ x^2 \text{ if } x > -2. \end{cases}$

(a) Is $f(x)$ continuous at $x = -5$?

$f(-5)=10$

$\lim_{x\to -5^+} f(x) = 10 = \lim_{x\to -5^-} f(x)$

so $\lim_{x\to -5} f(x) = 10$ and $\boxed{f \text{ is continuous at } x=-5}$

(b) Is $f(x)$ continuous at $x = -2$?

$f(-2)$ does not exist

so $\boxed{f \text{ is not continuous at } x=-2}$

(13) Find the derivative of $f(x) = 2x^2$.

① $f(x+h) = 2(x+h)^2 = 2x^2+4xh+2h^2$

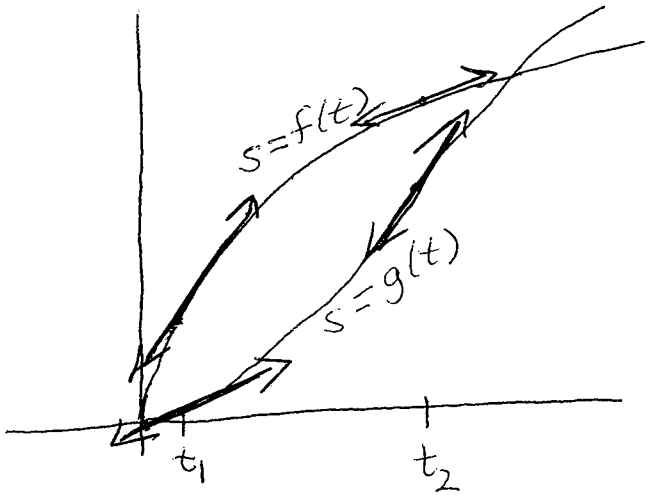
② $f(x+h)-f(x) = 2x^2+4xh+2h^2-2x^2$
$= 4xh+2h^2$

③ $\frac{f(x+h)-f(x)}{h} = \frac{4xh+2h^2}{h} = \frac{h(4x+2h)}{h}$
$= 4x+2h$

④ $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} 4x+2h = 4x$

so $\boxed{f'(x) = 4x}$

(14) The position of car A is given by $s = f(t)$ and the position of car B is given by $s = g(t)$.



(a) Which car is driving faster at t_1?

The slope is steeper on the curve $s=f(t)$ so Car A is faster at t_1.

(b) Which car is driving faster at t_2?

The slope is steeper on the curve $s=g(t)$ so car B is faster at t_2.