Homework 3

MATH 401

10.3 pf : Let $P(n)$ be the statement :

$$1^2 + 2^2 + \dots + n^2 = \frac{1}{6} n (n+1)(2n+1), \ \forall n \in \mathbb{N}$$

$P(1)$ asserts $1^2 = \frac{1}{6}(2)(3)$, which is true, and serves to establish the basis for induction.

Now assume $P(n)$ is true.

$P(n+1)$ asserts :

$$1^2 + 2^2 + \dots + n^2 + (n+1)^2$$

$$= \frac{1}{6} n(n+1)(2n+1) + (n+1)^2 \quad \dots \text{(by induction hypothesis)}$$

$$= \frac{1}{6}(2n^3 + 3n^2 + n) + n^2 + 2n + 1$$

$$= \frac{1}{6}(2n^3 + 9n^2 + 13n + 6)$$

$$= \frac{1}{6}[(n+1)(n+2)(2n+3)] = \frac{1}{6}[(n+1)(n+2)(2n+3)] \dots \text{(to clarify sloppiness)}$$

$$= \frac{1}{6}(n+1)((n+1)+1)(2(n+1)+1)$$

Thus, $P(n+1)$ is true whenever $P(n)$ is true

& by induction we conclude $P(n)$ is true for $\forall n \in \mathbb{N}$ ∎

10.4 pf : Let $P(n)$ be the statement : $1^3 + 2^3 + \dots + n^3 = \frac{1}{4} n^2 (n+1)^2, \ \forall n \in \mathbb{N}$

$P(1)$ asserts $1^3 = \frac{1}{4}(2)^2$, which is true

Now assume $P(n)$ is true.

$P(n+1)$ asserts :

$$1^3 + 2^3 + \dots + n^3 + (n+1)^3 = \frac{1}{4} n^2 (n+1)^2 + (n+1)^3 \quad \text{(by induction hypothesis}$$

$$= \frac{1}{4}[n^2(n^2 + 2n + 1)] + n^3 + 3n^2 + 3n + 1$$

$$= \frac{1}{4}[n^4 + 2n^3 + n^2] + n^3 + 3n^2 + 3n + 1$$

$$= \frac{1}{4}[n^4 + 6n^3 + 13n^2 + 12n + 4]$$

$$= \frac{1}{4}[(n+1)^2 (n+2)^2]$$

$$= \frac{1}{4}[(n+1)^2 ((n+1)+1)^2]$$

Thus, $P(n+1)$ is true whenever $P(n)$ is true

& by induction we conclude $P(n)$ is true for $\forall n \in \mathbb{N}$ ∎