0.14 Pf : Let $P(n)$ be the statement $9^n - 4^n$ is a multiple of 5 for all $n \in \mathbb{N}$, i.e. $9^n - 4^n = 5m$

$P(1)$ asserts $9^1 - 4^1 = 5(1)$, which is true

Now, Assume $P(n)$ is true.

$P(n+1)$ asserts $9^{n+1} - 4^{n+1}$ is a multiple of 5

Note that
$$
\begin{aligned}
9^{n+1} - 4^{n+1} &= 9(9^n) - 4(4^n) \\
&= 9(9^n - 4^n + 4^n) - 4(4^n) \\
&= 9(5m + 4^n) - 4(4^n) \quad \ldots \text{(by induction hypothesis)} \\
&= 45m + 9(4^n) - 4(4^n) \\
&= 45m + 5(4^n) \\
&= 5(9m + 4^n)
\end{aligned}
$$

Thus, $9^{n+1} - 4^{n+1}$ is a multiple of 5

$\therefore$ $P(n+1)$ is true whenever $P(n)$ is true

$\therefore$ by induction we conclude $P(n)$ is true for all n ⊠

0.20 pf : Let $P(n)$ be the statement

$$(\cos x + i \sin x)^n = \cos(nx) + i \sin(nx), \quad \forall n \in \mathbb{N}, \; i = \sqrt{-1}$$

$P(1)$ asserts that $(\cos x + i \sin x)^1 = \cos(1 \cdot x) + i \sin(1 \cdot x)$, which is true

Now, assume $P(n)$ is true

$P(n+1)$ asserts :

$$
\begin{aligned}
(\cos x + i \sin x)^{n+1} &= (\cos x + i \sin x)^n (\cos x + i \sin x) \\
&= [\cos(nx) + i \sin(nx)](\cos x + i \sin x) \quad \ldots \text{(by induction hypothesis)} \\
&= \cos(nx)\cos x - \sin(nx)\sin(x) + \ldots \\
&\quad \ldots + i\left(\cos(nx)\sin x + \sin(nx)\cos x\right) \\
&= \cos(nx + x) + i \sin(nx + x) \quad \ldots \text{(by identities given in book)} \\
&= \cos((n+1)x) + i \sin((n+1)x)
\end{aligned}
$$

Thus, $P(n+1)$ is true whenever $P(n)$ is true

$\therefore$ by induction we conclude $P(n)$ is true for all n ⊠