11.8 $P = \{x \in \mathbb{R} \mid x > 0\}$

a) Show if $x, y \in P$ then $x+y \in P$

If $x, y \in P$, then $x > 0$ and $y > 0$.
By O3) $x+y > 0+y = y > 0$. Thus $x+y \in P$.

b) Show if $x, y \in P$ then $xy \in P$.
If $x, y \in P$ then $x > 0$ and $y > 0$
By O4) $x \cdot y > 0 \cdot y = 0$, the last equality follows by Theorem (11.1 b). Thus $xy \in P$.

c) For each $x \in \mathbb{R}$, show $x \in P$, $x = 0$ or $-x \in P$.
By trichotomy, 1) $x > 0$, 2) $x = 0$ or 3) $x < 0$.
In the first case, $x \in P$.
In the second case, $x = 0$.
In the third case, $x < 0$ and by 11.1e
$-x > -0 = -1 \cdot 0$ (by 11.1 e)
Now by 11.1 b $-1 \cdot 0 = 0$. Thus $-x > 0$
which implies $-x \in P$.