.9

01 for all $x, y \in P$, exactly one of the following is true:

$x = y$, $x > y$, or $x < y$

~~For "="~~ For "$<$"

Exactly one of the following is true:

① $x - y \in P \Rightarrow y < x$

② $y - x \in P \Rightarrow x < y$

③ $x - y = 0 \in P \Rightarrow x = y$

} 11.8 ~~(a)~~ (c)

02 For all $x, y, z \in P$, if $x < y$ and $y < z$, then $x < z$

if $x < y$, then $y - x \in P$ ... (1)

if $y < z$, then $z - y \in P$ ... (2)

} by definition

Adding (1) and (2) (possible b/c of 11.8 (a)) ~~yields~~

~~$z - x \in P$~~ ~~$y + z$~~

$(y - x) + (z - y) \in P$

$(y - y) + (z - x) \in P$ ... A3

$z - x \in P$ .... A5

$\Rightarrow x < z$ ... by defn.

03 For all $x, y, z \in P$, if $x < y$, then $x + z < y + z$

$x < y \iff y - x \in P$ ... by definition

$y - x + (z - z) \in P$ ... A5 and A4

$(y + z) - (x + z) \in P$ ... A3

$\Rightarrow x + z < y + z$ ... by definition