Probability Notes
for Janet Vassilev's Math 317 course

A *sample space* is the set of all possible outcomes of an experiment. Since we are talking about counting finite sets, our sample spaces will be finite, but in general the sample space need not be finite. A subset of a sample space is called an *event*. If we want to find the probability of an event A inside a sample space S, we compute $P(A) = \frac{|A|}{|S|}$. Note that $0 \le P(A) \le 1$ where $P(A) = 1$ if and only if $A = S$ and $P(A) = 0$ if and only if $A = \emptyset$.

If $A_1, \ldots, A_n$ is a partition of an event A, then we use the addition principal to obtain that $P(A) = P(A_1) + \cdots + P(A_n)$.

Similarly, we can use the subtraction principle to obtain $P(A^c) = 1 - P(A)$.

If $A_1, \ldots, A_n$ are independent events in sample spaces $S_1, \ldots, S_n$, then the probability that all occur is given by the multiplication principle and the formula is $P(A_1 \times \cdots \times A_n) = P(A_1) \cdots P(A_n)$.

Suppose A and B are events in a sample space S, we can define the conditional probability that B will occur given that A has occurred, this is given by the formula $P(B \mid A) = \frac{P(A \cap B)}{P(A)}$.

Here is an example:

Suppose the sample space is the set of rolls of two distinct dice. Find the condition probability that the sum of the rolls is at least 8 given that the sum of the rolls is at most 9.

$B = \{(2,6),(3,5),(4,4),(5,3),(2,6),(3,6),(4,5),(5,4),(6,3),(4,6),(5,5),(6,4),(5,6),(6,5),(6,6)\}$ and $A^c = \{(4,6),(5,5),(6,4),(5,6),(6,5),(6,6)\}$. Thus

$$A \cap B = \{(2,6),(3,5),(4,4),(5,3),(2,6),(3,6),(4,5),(5,4),(6,3)\}.$$

Thus $P(B \mid A) = \frac{\frac{9}{36}}{\frac{30}{36}} = \frac{3}{10}$.