

RESEARCH STATEMENT

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My research lies in the realm of commutative ring theory. One of my interests is to study new algebraic structures determined by various closure operations. I am also pursuing the determination of which finiteness properties hold for local cohomology modules $H_I^i(R)$ depending on the type of the ring (regular local ring, Gorenstein local ring, etc.) and the height of the ideal. Another emphasis is the study of singularities determined by quotients of regular rings. Below I describe my results and future plans.

1. NEW STRUCTURES DEFINED BY CLOSURE OPERATIONS

Closure operations have become an important tool for classifying the singularities of the ring. For example, the algebraic formulation of the Briançon-Skoda theorem involves integral closure. Also the development of tight closure, in the late eighties by Hochster and Huneke gave a slick proof of the Briançon-Skoda theorem and generalized the theorem to non-regular rings.

Most commutative rings have a multitude of closure operations defined on them. Closure operations are operations, cl , defined on the set of ideals of a ring satisfying a) $I \subseteq I^{cl}$, b) $I^{cl} \subseteq J^{cl}$ if $I \subseteq J$ and c) $(I^{cl})^{cl} = I^{cl}$. Tight closure and integral closure are closure operations, but there are other interesting closure operations defined on local rings such as basically full closure and \mathfrak{m} -full closure (described below). In trying to understand the relationship between these closure operations, I looked into the algebraic structure on the set of all closure operations in [Va3] thinking of closure operations as maps from the set of ideals of a ring to itself. The set of maps from the ideals of a ring to itself is a monoid under composition. Over a discrete valuation ring, I have exhibited examples of two closure operations which when composed with each other do not form a closure operation on the ring. When we add the additional property d) $I^{cl}J^{cl} \subseteq (IJ)^{cl}$ which makes the closure operation a semiprime operation, the algebraic structure is not a monoid, but it decomposes into the union of two monoids where one is a left act of the other for both a DVR, a PID and excluding some exceptional semiprime operations this is also true over the ring $K[[x^2, x^3]]$. A prime operation is a semiprime operation satisfying e) $bI^{cl} = (bI)^{cl}$ for all regular elements $b \in R$. In [Va4] I have shown in all semigroup rings where all ideals are generated by at most two elements then the only prime operation is the identity and have exhibited that this is not the case in a semigroup ring which has ideals that are generated by at least three elements. Trying to better understand the semiprime and prime operations on one dimensional rings is a project I am still pursuing along with my graduate students Laurie Price and Bryan White.

Two notions of fullness, were mentioned above in terms of closures related to them: \mathfrak{m} -full and basically-full. \mathfrak{m} -full ideals were first introduced by Rees and written up by Watanabe [WaJ]. An ideal I is \mathfrak{m} -full if there exists an x such that $(\mathfrak{m}I : x) = I$. Ideals which are \mathfrak{m} -full satisfy the Rees Property: $\mu(J) \leq \mu(I)$ for all $J \supseteq I$. In 2-dimensional regular local

rings the ideals I satisfying the Rees Property are precisely those ideals which are \mathfrak{m} -full. A related concept is basically full introduced by Heinzer, Ratliff and Rush [HRR]. An ideal I is basically-full if for any ideal $J \supseteq I$, no minimal basis for I can be extended to a minimal basis of J . They have shown that basically full ideals are \mathfrak{m} -primary ideals and they satisfy $(\mathfrak{m}I : \mathfrak{m}) = I$. Recently, Hong, Lee, Noh and Rush [HLNR] showed that for parameter ideals in any regular local ring \mathfrak{m} -full and basically full are equivalent. I have used tight closure to generalize these concepts in rings of characteristic p . An ideal I is $*$ - \mathfrak{m} -full if $(\mathfrak{m}I : x) = I^*$. An \mathfrak{m} -primary ideal I is $*$ -basically full if $(\mathfrak{m}I : \mathfrak{m}) = I^*$. If R is weakly F -regular, the tight closure versions are precisely equal to their $*$ -less counterparts. However, if the ring is not weakly F -regular, there are examples of $*$ - \mathfrak{m} -full ideals which are not \mathfrak{m} -full and $*$ -basically full ideals which are not basically full. Heinzer, Ratliff and Rush showed in [HRR] that R is a PID if and only if every \mathfrak{m} -primary ideal is basically full. Vraciu and I [VV] have extended the notion of $*$ -basically-full to the notion of $*$ - T -basically full: $(TI : T) = I^*$ for any ideal T of R .

Theorem 1.1. *Let (R, \mathfrak{m}) be a complete local Cohen Macaulay normal domain of positive characteristic and let τ be the test ideal. If T is an ideal of grade at least two, then every \mathfrak{m} -primary ideal is $*$ - T -basically full if and only if R is weakly F -regular and $T = R$. In particular, every \mathfrak{m} -primary ideal is $*$ - τ -basically full if and only if R is weakly F -regular.*

Then as a corollary, we get the following extension of Heinzer, Ratliff and Rush's Theorem:

Theorem 1.2. *Let (R, \mathfrak{m}) be a local Cohen Macaulay domain of characteristic p . (a). If R is a one-dimensional ring with test ideal equal to \mathfrak{m} , then all \mathfrak{m} -primary ideals are $*$ -basically full. (b). Assume in addition that R is normal and has perfect residue field. Then R is a one-dimensional ring if and only if all \mathfrak{m} -primary ideals are $*$ -basically full.*

Louiza Fouli and I [FV] defined the cl -core of an ideal, for a closure operation cl to be the intersection of all cl -reductions of an ideal. In general the cl -core can be much larger than the core itself. In the case of tight closure, we have shown that if $R = \mathbb{Z}/p\mathbb{Z}(u, v, w)[[x, y, z]]/(ux^p + vy^p + wz^p)$, $\text{core}(\mathfrak{m}^2) \subsetneq *-\text{core}(\mathfrak{m}^2)$. However, if the $*$ -spread of an ideal is equal to the spread then we were able to show the following.

Theorem 1.3. *Let R be a local Cohen Macaulay normal domain of characteristic $p > 0$ with perfect infinite residue field. Let I be an ideal with $\ell^*(I) = \ell(I) = s$. We further assume that I satisfies G_s and is weakly $(s - 1)$ -residually S_2 . Then $\text{core}(I) = *-\text{core}(I)$.*

In [FVV], Fouli, Vraciu and I have found the formula for the $*$ -core of I to be $J(J : I)$, for some $*$ -reduction J when I is the tight closure of an ideal generated by a system of parameters or when I is the tight closure of a sufficiently high Frobenius power of an ideal generated by $*$ -independent elements. In both of these scenarios, the $*$ -reduction number of I was one. We believe a more general formula for the $*$ -core will be $J^r(J : I)$ where r is the $*$ -reduction number of the ideal with respect to the minimal $*$ -reductions.

2. FINITENESS PROPERTIES OF LOCAL COHOMOLOGY

Let R be a Noetherian ring and M be a finitely generated module. Finiteness properties of R and M are abundant. All ideals of R are finitely generated. All submodules of M are finitely generated. R and M both have a finite number of associated primes. $\text{Ext}_R^i(R/I, M)$ are finitely generated for all I and i . And the list keeps going.

Then enters $H_I^n(M)$, the n th local cohomology module of M with respect to I , introduced by Grothendieck in the 1960's. The local cohomology modules, $H_I^n(M)$, were more mysterious, in fact they can be highly non finitely generated. Much theory was developed about the local cohomology modules, but examples were few and far between. One of the most quoted examples was Hartshorne's 1970 example [Ha]. He showed if $R = k[[x, y]][u, v]$, $M = R/(xu + yv)$ and $I = (u, v)$, then $\text{Soc}(H_I^2(M))$ and hence $\text{Hom}(R/I, H_I^2(M))$ were infinitely generated. But some were still wondering if local cohomology modules could have other finiteness properties.

In 1990, Huneke gave a talk [Hu1] on several finiteness problems and conjectures regarding local cohomology. Since then many have made a concerted effort to affirm or disaffirm these conjectures. For regular local rings (R, m) , work of Huneke and Sharp in characteristic p [HSh] and Lyubeznik [L1] [L2] in characteristic 0 and mixed characteristic over an unramified regular local ring have shown the Bass numbers of the local cohomology modules $H_I^n(R)$ are finite and the set of associated primes to $H_I^n(R)$ is finite. Marley showed in [M] if $\dim(M) \leq 3$ then $H_I^i(M)$ has a finite number of associated primes for all i and I . Then in [MV1] Marley and myself extended this result for all modules M over a 4 dimensional regular local ring and for modules M and ideals I satisfying $\dim(M/IM) \leq 2$ and satisfying Serre's criterion S_3 .

Around the same time Marley's original result for modules of dimension three or less, Singh exhibited the following example in [Si]: $R = \mathbb{Z}[x, y, z, u, v, w]/(xu + yv + zw)$ and $I = (x, y, z)$, $H_I^3(R)$ has an infinite number of associated primes. Later using graded techniques, Katzman found in [Kat], an example of a local Noetherian ring with a local cohomology module with an infinite number of associated primes: Let $R = k[x, y, s, t, u, v]$, $m = (x, y, s, t, u, v)$, $S = R_m$ and $I = (u, v)$ and $f = sx^2v^2 - (s + t)xyuv + ty^2u^2$. Then $H_I^2(S/fS)$ has an infinite number of associated primes. Singh and Swanson [SS] produce an infinite class of similar examples. These local cohomology modules with an infinite number of associated primes appear as top graded local cohomology modules. Katzman and Sharp have exhibited in [KS] that although Katzman's and Singh's examples do have an infinite number of associated primes, they have a finite number of minimal associated primes. One of my interests is showing that all local cohomology modules have a finite number of minimal associated primes.

Cofiniteness of a module is a stronger property than the finiteness of its associated primes. A question considered by Hartshorne in 1970 in his oft quoted paper [Ha], "Affine Duality and Cofiniteness, "Do the A -modules which are J -cofinite form a subcategory of all A -modules?" Certainly, the answer may depend on the ideal J considered. The answer is false considering the example he constructed in this same paper: Let $A = k[x, y, z, w]$, $J = (x, y)$. Note that $H_J^2(A)$ is J -cofinite, but $H_J^2(A/(xz + yw))$ is not J -cofinite. Hartshorne, himself, showed for certain ideals, J , namely $J = (f)$, a principal ideal or J a dimension one prime ideal both contained in a complete regular local ring A , that the J -cofinite modules do indeed form a subcategory of the category of A -modules. Delfino and Marley [DM] extended his result to dimension one primes over a complete local ring and believe the result should hold for arbitrary dimension one ideals over an arbitrary local ring. Melkersson [Me2] also extended the result for any ideal over a two-dimensional ring. However, Melkersson also has an example in [Me1] of a three-dimensional regular local ring $A = k[[x, y, z]]$, and an ideal $J = (xz, yz)$ of height one and dimension A/J is two where $H_J^2(A)$ is not J -cofinite. This seems to indicate that when the dimension of the ring A is greater than two, the set

of modules which are J -cofinite, where J has prime components of varying heights, might not form a subcategory of all A modules.

One important consequence of a set of modules being a subcategory of the category of R -modules for a local ring R , is that for any module in the set, all its submodules and homomorphic images are also in the set. Although, the J -cofinite modules of a local ring R may not form a subcategory of the category of R -modules, there may be subsets of the set of J -cofinite modules which behave well, i.e. within the subset all submodules and homomorphic images remain J -cofinite. For example, Melkersson [Me1] shows that every submodule and quotient of an artinian J -cofinite module is still J -cofinite. So, it may be the case, that there are bigger subsets of the set of J -cofinite modules for which submodules and quotients remain J -cofinite.

Marley and myself observed in [MV1] that $\{x \in R \mid M_x \text{ is a finitely generated } R_x\text{-module}\}$ is an ideal. Hence, $C(M, J) = \{x \in R \mid M_x \text{ is } J_x \text{ cofinite}\}$ is an ideal and $V(C(M, J))$ represents the non J -cofinite locus of M . Hence, J -cofiniteness is an open property. Note, for Melkersson's example, for all $p \neq (x, y, z)$, $H_J^2(A)_p$ is J_p -cofinite since $\dim A_p \leq 2$. In this case, the submodules and quotients of all J_p -cofinite modules remain J_p cofinite. We know that $H_J^2(A)$ is not J -cofinite but what if for some other module A -module M , we know that M is J -cofinite and M_p is J_p -cofinite for all p . Can we see that the submodules and quotients of M will also be J -cofinite?

In addition to showing that cofiniteness is an open property, Marley and myself also prove in [MV1] the following generalization of [HK, Theorem 3.6(ii)]:

Theorem 2.1. *Let (R, m) be a complete Cohen-Macaulay normal local ring and I an ideal such that $\dim R/I \geq 2$ and $\text{Spec } R/I - \{m/I\}$ is disconnected. Then $\text{Hom}_R(R/I, H_I^{d-1}(R))$ is not finitely generated. Consequently, $H_I^{d-1}(R)$ is not I -cofinite.*

Related to the question of when $H_J^i(M)$ is J -cofinite, is when is $\text{Hom}_R(R/m, H_J^i(M))$, the socle of $H_J^i(M)$, finitely generated. Determining if the socle of $H_J^i(M)$ is finitely generated and $\text{Supp}_R H_J^i(M) \subseteq \{m\}$ is equivalent to showing that $H_J^i(M)$ is Artinian. Hartshorne's example is an example of a local cohomology module with infinitely generated socle. Since Hartshorne gave his example over thirty years ago, the only progress on the question of when a local cohomology module has an infinitely generated socle has come from Helm and Miller [HM] who gave necessary and sufficient conditions on the semigroup for a ring to have a local cohomology module with a socle which is infinitely generated. Using graded techniques, Marley and I have been able to exhibit an infinite class of Noetherian local rings which have local cohomology modules which have infinite dimensional socles in [MV2].

Theorem 2.2. *Let (T, m) be a Noetherian local ring of dimension at least two. Let $R = T[x_1, \dots, x_n]$ be a polynomial ring in n variables over T , $I = (x_1, \dots, x_n)$ and $f \in R$ be a homogeneous polynomial whose coefficients form a system of parameters for T . Then the socle of $H_I^n(R/fR)$ is infinite dimensional.*

Marley and I are currently working on showing the mixed characteristic version of the following theorem of Huneke and Koh in characteristic p [HK] Lyubeznik in characteristic 0 [L1] that in the case of a regular ring $\text{Hom}(R/I, H_I^n(R))$, is finitely generated for $n \geq \max \{\text{ht}(P) \mid P \in \min(I)\}$ if and only if $H_I^n(R) = 0$. We can show for an unramified regular local ring $\text{Ann}_R(H_I^i(R))$ is 0 or all of R . We have defined the new concept of strongly I -cofinite, i.e., $\text{Hom}(R/I, H_I^n(M))$ is finitely generated for all n , then M is strongly I -cofinite.

If M is strongly I -cofinite, we are able to show that $H_I^n(M)$ is I -cofinite for all n . We have also come up with an example of a ring R , where the $H_P^n(R)$ is not P -cofinite and P is a minimal prime of R . No such example was known previously.

3. SINGULARITIES IN QUOTIENTS OF REGULAR RINGS

Most of my interest in studying singularities defined by quotients of regular rings has been through my interest in tight closure which one may think of as “characteristic p methods in commutative algebra” following Bruns [Br]. Since its invention by Hochster and Huneke in the eighties, tight closure has attacked many problems in both commutative algebra and algebraic geometry including: the “homological conjectures”, Big Cohen-Macaulay Modules, singularity theory, the Briançon-Skoda Theorem and more. Thus far no tight closure theory is known for rings of mixed characteristic, thus we require all rings that we work with contain a field.

Although tight closure is defined for rings of characteristic 0 containing a field, the characteristic p notion is easier to define. Let R be a commutative Noetherian ring of characteristic $p > 0$ and I be an ideal of R with generators x_1, \dots, x_n . Denote powers of p by q . Define R° to be the complement of the union of minimal primes. We say $x \in R$ is in the *tight closure* of I if there exists a $c \in R^\circ$ such that $cx^q \in I^{[q]}$ for all large q , where $I^{[q]} = (x_1^q, \dots, x_n^q)$. Denote the tight closure of I as I^* .

From the very definition of tight closure, we recognize the need for elements not contained in the union of minimal primes to compute tight closure. If such an element multiplies every element in the tight closure of an ideal into the ideal itself for all ideals in R we call this element a test element. The ideal these test elements generate, the test ideal, is an object present in most of my research interests in the area. Test elements and test ideals play a key role in tight closure theory. Knowing test elements allows us to compute tight closures of ideals, prove persistence of tight closure, in otherwords if $\phi : R \rightarrow S$ be a homomorphism of F -finite reduced Noetherian rings then if $x \in I^*$ then $\phi(x) \in (IS)^*$, and in the case where R is a Gorenstein or \mathbb{Q} -Gorenstein isolated singularity we can determine the singularity type of the ring R . The test ideal in a Gorenstein or a \mathbb{Q} -Gorenstein ring is defined to be the annihilator of a submodule of the local cohomology module $H_m^d(\omega_R)$. My research focuses on the test ideals in two situations: 1) when the ring is F -finite and a reduced quotient of a regular local ring, and 2) when the ring is a Gorenstein isolated singularity.

Recall a ring R is F -finite if $F(R)$ is a finite R -module. Let S be an F -finite regular local ring and I an ideal contained in S . Define $R = S/I$. In [Fe] Fedder found a nice criterion to check whether such a ring is F -pure, i.e. if $IF(R) \cap R = I$. It states such a ring is F -pure if and only if $(I^{[p]} : I) \not\subseteq \mathfrak{m}^{[p]}$. Even if $I \subseteq J$ we don't necessarily know that $(I^{[q]} : I) \subseteq (J^{[q]} : J)$. But in such a ring, I have shown in [Va1] the following:

Theorem 3.1. *Let $R = S/I$, where S is a regular local ring. Let τ_R be the pull back of the test ideal of R in S . Then $(I^{[q]} : I) \subseteq (\tau_R^{[q]} : \tau_R)$.*

Recently, Smith and Lyubeznik have generalized the above theorem to non-local complete reduced rings in [LS]. Now if (R, \mathfrak{m}) is F -pure then $(I^{[p]} : I) \not\subseteq \mathfrak{m}^{[p]}$ implies that $(\tau_R^{[p]} : \tau_R) \not\subseteq \mathfrak{m}^{[p]}$ as a consequence of the above theorem. To see the test ideal is nonzero we use theorem of Hochster and Huneke [HH1] which states that if an F -finite reduced ring R of characteristic p has a nonzero c such that R_c is regular. Then c has a power which is a test element. Hence, $\text{ht}(\tau) \geq 1$. Define $R_1 = R/\tau_R$. Note that R_1 is reduced by a theorem of

Fedder and Watanabe [FW] and F -pure by Fedder's F -purity criterion [Fe]. Again by the above, $(\tau_R^{[p]} : \tau_R) \subseteq (\tau_{R_1}^{[p]} : \tau_{R_1})$. Thus $R_2 = R/\tau_{R_1}$ is F -pure. Continuing on in this fashion we get the filtration $I \subseteq \tau_R \subseteq \tau_{R_1} \dots \subseteq \tau_{R_n} \subseteq \dots$ in which all of the R_n are F -pure and $\text{ht}(\tau_{R_{i+1}}/\tau_{R_i}) \geq 1$.

To find examples of these filtrations I have made explicit calculations of test ideals in polynomial and power series rings modulo monomial ideals.

Example 3.2. Let $T = k[[x_1, \dots, x_n]]$ or $T = k[x_1, \dots, x_n]$. Set $R = T/I$ where $I = \langle x_{i_1} \dots x_{i_d} \rangle_{1 \leq i_1 < \dots < i_d \leq n}$, then $\tau = \langle x_{i_1} \dots x_{i_r} \dots x_{i_d} \rangle_{1 \leq i_1 < \dots < i_d \leq n}$.

Thus the τ_i/τ_1 for $i > 1$ should be isomorphic to appropriate ideals contained in $(x_{i_1} \dots x_{i_s} | i_1 < \dots < i_s)$.

I believe that the rings

$$k[[x_1, x_2, \dots, x_r]] / \langle x_{i_1} \dots x_{i_{s+1}} | 1 \leq i_1 < \dots < i_{s+1} \leq r \rangle$$

will play a key role in the classification of s dimensional complete F -pure rings as well as

$$k[[x_{11}, \dots, x_{1t_1}, \dots, x_{rt_r}]] / \langle x_{j_{i_1}} \dots x_{j_{i_j}} | 1 \leq j \leq r, 1 \leq i_1 < \dots < i_{k_j} \leq t_j \rangle$$

with $k_1 + k_2 + \dots + k_r = s + r$. Although these rings will be among the F -pure rings of dimension s , there should be more, but the ideals defining the F -pure rings of dimension two or more will vary depending on the characteristic of k .

In my thesis [Va2], I classified all Gorenstein double points with test ideal equal to the maximal ideal. The classification exhibits that such singularities are minimally elliptic [La]. I am interested in further understanding the relationship between singularities and their test ideals.

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