MATH 311 – REVIEW FOR EXAM 2

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1. Cartesian, Cylindrical and Spherical coordinates

Note: You may assume that the formulas for ∇f, $\nabla \times \mathbf{F}$, $\nabla \cdot \mathbf{F}$ in cylindrical and spherical coordinates are given to you. Be able to:

- Compute ∇f, $\nabla \times \mathbf{F}$, $\nabla \cdot \mathbf{F}$ in any of the three coordinate systems.
- Use formulas to find $\Delta f = \nabla^2 f = \nabla \cdot (\nabla f)$ in any of the three coordinate systems.

Good Problems: §3.10: 6,7,8,10,11,12, §3.6: 7, §3.5: 4

2. Line Integrals

- Parametrize basic curves (portions of circles, ellipses, lines, helices) by $\mathbf{R}(t) = \big\langle x(t), y(t), z(t)\big\rangle, t \in [a, b]$
- Evaluate line integrals of the form $\int_C f(x, y, z)ds$ (such as arclength, where $f = 1$) using the formula

$$ds = \left|\frac{d\mathbf{R}}{dt}\right| dt = \left|\left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right\rangle\right| dt = \sqrt{\frac{dx}{dt}^2 + \frac{dy}{dt}^2 + \frac{dz}{dt}^2}\, dt$$

- Evaluate line integrals of the form $\int_C \mathbf{F}\cdot\mathbf{T}\, ds = \int \mathbf{F}\cdot\mathbf{ds} = \int_C Pdx + Qdy + Rdz$ (note that C needs to be oriented here) using the formula

$$\mathbf{ds} = \frac{d\mathbf{R}}{dt}\, dt = \left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right\rangle dt$$

Good Problems: 200: find arclength of helix in §3.5: 10, $0 \le t \le 2$, 201: find $\int_C x\, ds$ where C is helix in previous problem, §4.1: 1,2,3,4,8,14,19.

3. Surface Integrals

- Parametrize basic surfaces (portions of planes, cylinders, cones, spheres) by $\mathbf{R}(u, v) = \big\langle x(u, v), y(u, v), z(u, v)\big\rangle, (u, v) \in D$
- Evaluate surface integrals of the form $\int_S f(x, y, z)\, dS$ (such as surface area, where $f = 1$) using the formula

$$dS = \left|\frac{\partial\mathbf{R}}{\partial u} \times \frac{\partial\mathbf{R}}{\partial v}\right| du\, dv$$

- Evaluate surface integrals of the form $\int_C \mathbf{F}\cdot\mathbf{n}\, dS = \int \mathbf{F}\cdot\mathbf{dS}$ (note that S needs to be oriented here) using the formula

$$dS = \frac{\partial\mathbf{R}}{\partial u} \times \frac{\partial\mathbf{R}}{\partial v}\, du\, dv$$

- Evaluate surface integrals of the form $\int_C \mathbf{F}\cdot\mathbf{n}\, dS = \int \mathbf{F}\cdot\mathbf{dS}$ by inspection, in those cases where $\mathbf{F}\cdot\mathbf{n}$ simplifies to a constant. Here, you need to be able to find the unit normal $\mathbf{n}$ for basic surfaces (cylinders, spheres, cubes, planes, and surface give by $F(x, y, z) = 0$).

Good Problems: 109,110,111,112, §4.7: 1,3,7c,10,11,

4. Conservative (irrotational) **and solenoidal** (incompressible) **vector fields**

Conservative $\mathbf{F}$: Irrotational $\nabla \times \mathbf{F} = 0$ (if D simply connected) $\Leftrightarrow$ there exists $f(x, y)$ such that $\nabla f = \mathbf{F}$ and $\int_C \mathbf{F} \cdot \mathbf{ds} = f(B) - f(A)$

Note: the last item implies that line integrals of conservative fields are path independent. The function f is called the potential function for $\mathbf{F}$. The level curves of f are normal to the vector field $\mathbf{F}$.

Solenoidal $\mathbf{F}$: Incompressible $\nabla \cdot \mathbf{F} = 0$ $\Leftrightarrow$ In 2D: $\mathbf{F} = \langle F_1, F_2, 0 \rangle$ there exists $\psi(x, y)$ such that $\frac{\partial \psi}{\partial x} = -F_2, \frac{\partial \psi}{\partial x} = F_1$

Note: The function ψ is called the streamfunction for $\mathbf{F}$. The level curves of ψ are tangent to the vector field $\mathbf{F}$. Be able to:

- Check whether a field is conservative and/or solenoidal by checking whether $\nabla \times \mathbf{F} = 0$ and $\nabla \cdot \mathbf{F} = 0$. (Alternatively, you could try to find the potential function or the streamfunction, but that is typically much harder. If the domain D has a hole however, this is the only possible way to show that a field is conservative.)
- Find potential function for conservative fields. Use it to evaluate line integrals if necessary.
- Find streamfunction for solenoidal fields.
- Plot level curves of potential functions, of streamfunction, in relation to the vector field.

Good problems: §4.3: 1,4,5,6,7, Quiz#2, §4.4: 6,10

5. Volume Integrals

Good problems: §4.8: 1,3,6

6. Divergence Theorem

- Know the statement of the theorem: $\iiint_E \nabla \cdot \mathbf{F} \, dV = \iint_S \mathbf{F} \cdot \mathbf{n} \, dS$ where $\mathbf{F}$ is a continuously differentiable vector field, S is the boundary of E.
- Be able to verify the theorem for given examples (compute both sides).
- Use it to rewrite surface integral as a volume integral or viceversa, when convenient.
- Use it to compute surface integrals in the special case when $\nabla \cdot \mathbf{F} = 0$ except at a point.

Good problems: 116,117, §4.9: 4,5,6 §5.1: 8,10, Verify the Divergence Theorem for the integrals in §4.9: 1,2,3(b)