

TOPICS COVERED (Sections 1.1-1.4,2.1-2.2,3.1-3.6)

1. Solutions to $A\mathbf{x} = \mathbf{b}$

Reduce $[A|\mathbf{b}]$ to Row Echelon Form or Reduced Row Echelon Form

Determine whether 1, none, or infinitely many solutions

Determine free variables and lead variables

Find solutions, also if multiple right hand sides.

What does it mean for $A\mathbf{x} = \mathbf{b}$ to be consistent?

Number of solutions

What are the possible number of solutions to $A\mathbf{x} = \mathbf{b}$ if $A_{n \times n}$ has n linearly independent columns?

*What are possible number of solutions to $A\mathbf{x} = \mathbf{b}$ if $A_{m \times n}$ has more columns than rows?
To $A\mathbf{x} = \mathbf{0}$?*

*What are possible number of solutions to $A\mathbf{x} = \mathbf{b}$ if $A_{m \times n}$ has more rows than columns?
To $A\mathbf{x} = \mathbf{0}$?*

2. Inverses of $A_{n \times n}$

Find them, given A .

Know formula for $n = 2$ by heart.

3. Compute determinants of $A_{n \times n}$

Be able to use either method:

Elimination

Cofactor expansion

Special cases

What is determinant if one row or column consists only of zeros?

What is determinant if one row or column is a multiple of another?

How does the determinant of a matrix change if an elementary row operation is applied to it?

4. Vector spaces

Know the basics!

How do we check if a subset of a vector space is a subspace?

Define what it means for n vectors to be linearly independent.

Define what it means for n vectors to be a spanning set for a subspace S .

Define what it means for n vectors to be a basis for a subspace S .

Be able to

Check whether a set of vectors is linearly independent (p.154:2,4,6)

Check whether a set of vectors span a given space (p.143:9,10,14)

Find space spanned by given set of vectors (for example, by finding a basis for and the dimension of the space). (do for vectors in p.180:2)

Check whether given vector is in span of set of vectors. (p.143:11)

Find basis for space, and its dimension. (p.162:11)

Special subspaces

Find the nullspace of a matrix. Find a basis for it, and dimension.

Find the row space of a matrix. Find a basis for it, and dimension.

Find the column space of a matrix. Find a basis for it, and dimension.

Rank of a matrix.

Definition

How do we find it?

Relation between rank and $\dim(N(A))$

Change of basis

Find the transition matrix from any one basis to any other (whether one is the standard basis or not) show all steps

Find the coordinates of a vector with respect to any basis.

5. Proofs: Be able to show/check simple facts.

Some Examples:

Is $(A^{-1}B^{-1})^{-1} = AB$?

How are $\det(A^{-1})$, $\det(A^T)$, $\det(\alpha A)$ related to $\det(A)$?

How are $(A^T)^{-1}$, $(5A)^{-1}$ related to A^{-1} ?

Simplify $(A^T)^T$, $(A^{-1})^{-1}$.

For other examples, see your Matlab homework, p91:1, p121:1. Can you prove which of the given matrices/numbers are equal to each other?

Note: These types of question may appear as short answer questions or as TRUE/FALSE questions. Make sure to go through all the true/false sections in the "Chapter Tests". Be able to explain each answer!

Note: LU Factorization is not on this list.

SOME ADDITIONAL STUDY PROBLEMS

p.30:8, 12 (dont use back substitution, reduce to RRE instead, same thing, more efficient, cleaner)

p.63:19,23,24,

p.78:10

p.113:7

p.142:1,2,3,5

p.144:12, p.155:5

Is it possible for the plane $x = 1$ to be a 2-dimensional subspace of \mathbb{R}^3 ?