

## Instability Versus Equilibrium Propagation of a Laser Beam in Plasma

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We obtain, for the first time, an analytic theory of the forward stimulated Brillouin scattering instability of a spatially and temporally incoherent laser beam that controls the transition between statistical equilibrium and nonequilibrium (unstable) self-focusing regimes of beam propagation. The stability boundary may be used as a comprehensive guide for inertial confinement fusion designs. Well into the stable regime, an analytic expression for the angular diffusion coefficient is obtained, which provides an essential correction to a geometric optic approximation for beam propagation.

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Laser-plasma interaction has both fundamental interest and is critical for future experiments on inertial confinement fusion (ICF) at the National Ignition Facility (NIF) [1]. NIF's plasma environment, in the indirect drive approach to ICF, has hydrodynamic length and time scales of roughly millimeters and 10 ns, respectively, while the laser beams that traverse the plasma have a transverse correlation length,  $l_c$ , of a few microns, and coherence time  $T_c$  of roughly a few ps. These microscopic fluctuations induce corresponding small-scale density fluctuations, and one might naively expect their effect on beam propagation to be diffusive provided self-focusing is suppressed by small enough [2]  $T_c$ ,  $T_c \ll l_c/c_s$ , with  $c_s$  the speed of sound. However, we find that there is a collective regime of the forward stimulated Brillouin scattering (FSBS) [3] instability which couples the beam to transversely propagating low frequency ion acoustic waves. The instability has a finite intensity threshold even for very small  $T_c$  and can cause strong nonequilibrium beam propagation (self-focusing) as a result.

We present for the first time an analytic theory of the FSBS threshold in the small  $T_c$  regime. In the stable regime, an analytic expression for the beam angular diffusion coefficient,  $D$ , is obtained to lowest order in  $T_c$ , which is compared with simulation.  $D$  may be used to account for the effect of otherwise unresolved density fluctuations on beam propagation in a geometric optic approximation. This would then be an alternative to a wave propagation code [4] that must resolve the beam's correlation lengths and time, and therefore is not a practical tool for exploring the large parameter space of ICF designs. Knowledge of this FSBS threshold may be used as a comprehensive guide for ICF designs. The important fundamental conclusion is, for this FSBS instability regime, that even very small  $T_c$  may not prevent significant self-focusing. It places a previously unknown limit in the large parameter space of ICF designs.

We assume that the beam's spatial and temporal coherence are linked as in the induced spatial incoherence [5]

method, which gives a stochastic boundary condition at  $z = 0$  ( $z$  is the beam propagation direction) for the various Fourier transform components [6],  $\hat{E}$ , of the electric field spatiotemporal envelope,  $E$ ,

$$\begin{aligned} \hat{E}(\mathbf{k}, z = 0, t) &= |\hat{E}(\mathbf{k})| \exp[i\phi_{\mathbf{k}}(t)], \\ \langle \exp[i[\phi_{\mathbf{k}}(t) - \phi_{\mathbf{k}'}(t')]] \rangle &= \delta_{\mathbf{k}\mathbf{k}'} \exp(-|t - t'|/T_c). \end{aligned} \quad (1)$$

The amplitudes,  $|\hat{E}(\mathbf{k})|$ , are chosen to mimic that of actual experiments, as in the idealized "top hat" model of NIF optics:

$$|\hat{E}(\mathbf{k})| = \text{const}, \quad k < k_m; \quad |\hat{E}(\mathbf{k})| = 0, \quad k > k_m, \quad (2)$$

with  $1/l_c \equiv k_m \simeq k_0/(2F)$ ,  $F$  the optic  $f$  number (the ratio of the focal length of the lens divided by the lens diameter), and the average intensity  $\langle I \rangle \equiv \langle |E|^2 \rangle = I_0$  determines the constant. At electron densities,  $n_e$ , small compared to critical,  $n_c$ , and for  $F^2 \gg 1$ ,  $E$  satisfies [7]

$$\left( i \frac{\partial}{\partial z} + \frac{1}{2k_0} \nabla^2 - \frac{k_0 n_e}{2 n_c} \rho \right) E = 0, \quad \nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right). \quad (3)$$

$k_0$  is approximately the laser wave number in vacuum. The relative density fluctuation,  $\rho = \delta n_e/n_e$ , absent plasma flow and thermal fluctuations which are ignored here, propagates acoustically with speed  $c_s$ :

$$\begin{aligned} (R_0^{\rho\rho})^{-1} \ln(1 + \rho) \\ \equiv \left( \frac{\partial^2}{\partial t^2} + 2\tilde{\nu} \frac{\partial}{\partial t} - c_s^2 \nabla^2 \right) \ln(1 + \rho) = c_s^2 \nabla^2 I. \end{aligned} \quad (4)$$

$\tilde{\nu}$  is an integral operator whose Fourier transform is  $\nu k c_s$ , where  $\nu$  is the Landau damping coefficient.  $E$  is in thermal units defined so that in equilibrium the standard  $\rho = \exp(-I_0) - 1$  is recovered. The physical validity of Eqs. (3) and (4) as a model of self-focusing in plasma has been discussed previously [8–10]. If  $n_e/n_c$  is taken constant, there are three dimensionless parameters for  $\rho \ll 1$ :  $\nu$ ,  $\tilde{I}_0 \equiv (k_0/k_m)^2 (n_e/n_c) I_0/\nu$ , and  $\tilde{T}_c \equiv k_m c_s T_c$ .

Since Eq. (3) is linear in  $E$ , it may be decomposed, at any  $z$ , into a finite sum,  $E = \sum_j E_{\mathbf{m}_j}(\mathbf{x}, z, t)$ , where each term has a typical wave vector  $\mathbf{m}_j$ :  $E_{\mathbf{m}_j}(\mathbf{x}, z = 0, t) \sim \exp(i\mathbf{m}_j \cdot \mathbf{x})$ . Cross terms  $E_{\mathbf{m}_j} E_{\mathbf{m}'_j}^*$ ,  $\mathbf{m}_j \neq \mathbf{m}'_j$ , in the intensity, vary on the time scale  $\tilde{T}_c$  so that their effect on the density response, Eq. (4), is suppressed for  $\tilde{T}_c \ll 1$  (see detailed discussion in [11]). Similar consideration may be applied to general media with slow nonlinear response, including photorefractive media [12]. Then the right-hand side of Eq. (4) can be approximated as

$$c_s^2 \nabla^2 I = c_s^2 \nabla^2 \sum_j |E_{\mathbf{m}_j}|^2 = c_s^2 \nabla^2 \int d\mathbf{v} F(\mathbf{x}, \mathbf{v}, z, t). \quad (5)$$

$$F(\mathbf{x}, \mathbf{v}, z, t) = \int d\mathbf{r} \sum_{jj'} \delta_{\mathbf{m}_j, \mathbf{m}'_{j'}} E_{\mathbf{m}_j}(\mathbf{x} - \mathbf{r}/2, z, t) \times E_{\mathbf{m}'_{j'}}^*(\mathbf{x} + \mathbf{r}/2, z, t) e^{i\mathbf{v} \cdot \mathbf{r}} / (2\pi)^2 \quad (6)$$

is a variant of the Wigner distribution function which satisfies, as follows from Eq. (3),

$$\frac{\partial F}{\partial z} + 2\mathbf{v} \cdot \frac{\partial F}{\partial \mathbf{x}} - \frac{i}{\pi^2} \int [\hat{\rho}(-2[\mathbf{v} - \mathbf{v}'], z, t) \times \exp(-2i[\mathbf{v} - \mathbf{v}'] \cdot \mathbf{x}) - \hat{\rho}(2[\mathbf{v} - \mathbf{v}'], z, t) \times \exp(2i[\mathbf{v} - \mathbf{v}'] \cdot \mathbf{x})] F(\mathbf{x}, \mathbf{v}', z, t) d\mathbf{v}' = 0, \quad (7)$$

with boundary value  $F(\mathbf{x}, \mathbf{v}, z = 0, t) \equiv F_0(\mathbf{v}) = |\hat{E}(\mathbf{v})|^2$ . Here the unit of  $x$  is  $(1/k_0)\sqrt{n_c/n_e}$  and that of  $z$  is  $(2/k_0)n_c/n_e$ . Zero density fluctuation  $\rho = \partial\rho/\partial t = 0$  is an equilibrium solution of (4), (5), and (7), whose linearization admits solutions of the form  $\delta\rho \sim e^{\lambda z} \exp[i(\mathbf{k} \cdot \mathbf{x} - \omega t)]$ , for real  $\mathbf{k}$  and  $\omega$ , with

$$\tilde{\lambda} \equiv k_0 \lambda / k_m^2 = \frac{\tilde{k}(i\tilde{I}_0 - 2f)}{2\tilde{I}_0} \left[ \frac{f^2 \tilde{k}^2 - if\tilde{I}_0 \tilde{k}^2 - \tilde{I}_0^2}{f(f - i\tilde{I}_0)} \right]^{1/2}, \quad f \equiv \frac{\omega^2 - k^2 c_s^2 + 2i\nu\omega k c_s}{2i\nu k^2 c_s^2}, \quad \tilde{k} \equiv \frac{k}{k_m}. \quad (8)$$

Here and below, we assume that the principle branches of square and cubic roots are always chosen so that the branch cut in the complex plane is on the negative axis and values of square root and cubic root are positive for positive values of their arguments. The real part of  $\lambda$ ,  $\lambda_r \equiv \text{Re}(\lambda)$  has a maximum, as a function of  $\omega$ , close to resonance,  $\omega = \pm k c_s [1 + O(\nu)]$ . Below, we calculate all quantities at resonance  $\omega = \pm k c_s$ , because analytical expressions are much simpler in that case.  $\lambda_r(k)$  has a maximum,  $\lambda_{\max} = k_m^2 \tilde{\lambda}_{\max} / k_0 > 0$ , at  $k \equiv k_{\max}$ ,

$$k_{\max}/k_m = \tilde{I}_0 \sqrt{7(3\tilde{I}_0^2 - 2)2^{2/3} c^{-1} + 8 - 2^{1/3} c} [3^{1/2} 2(1 + \tilde{I}_0^2)^{1/2}]^{-1}, \quad c = (c_1 + c_2)^{1/3}, \quad (9)$$

$$c_1 = -40 + 225\tilde{I}_0^2 - 27\tilde{I}_0^4, \quad c_2 = -3i(\tilde{I}_0^2 + 4)\sqrt{27 - 60\tilde{I}_0^2 - 81\tilde{I}_0^4}.$$

Modes with  $k > k_{\text{cutoff}}$  are stable ( $\lambda_r < 0$ ), with  $k_{\text{cutoff}} = k_m \tilde{I}_0^2 (1 + \tilde{I}_0^2)^{-1/2} / 2$ , which defines a wave-number-dependent FSBS threshold.

As  $\tilde{I}_0 \rightarrow 0$ , at fixed  $k$ ,  $k_0 \lambda_r \rightarrow -k^2 / \tilde{I}_0$ , recovering the  $\delta(z)$  behavior of density response function  $R_0^{\rho\rho}$  in (4). If  $k_m$  is set to zero, the coherent FSBS convective gain rate [3] is recovered in the paraxial wave approximation. Unlike the static response  $\lambda(k, \omega = 0)$  which is stable [13] for all  $k$  for small enough  $I_0$ , the resonant response remains unstable at small  $k$  [14] since, as  $\tilde{I}_0 \rightarrow 0$ ,  $\tilde{\lambda}_{\max} \rightarrow 0.024\tilde{I}_0^5$  and  $k_{\text{cutoff}} \rightarrow k_m \tilde{I}_0^2 / 2$ .

Since the FSBS instability peaks near  $\omega = \pm k c_s$ , one expects an acousticlike peak to appear in the intensity fluctuation power spectrum,  $|I(k, \omega)|^2$ , for  $k$  less than  $k_{\text{cutoff}}$  as in the simulation ( $f/8$ ,  $n_e/n_c = 0.1$ ) results shown in Fig. 1. The fraction of power in this acoustic peak,  $\int_{2kc_s/3 < |\omega| < 4kc_s/3} |I(k, \omega)|^2 d\omega / \int_{-\infty}^{+\infty} |I(k, \omega)|^2 d\omega$ , increases significantly as  $\tilde{I}_0$  passes through its threshold value for a particular  $k$ , as shown in Fig. 2. There is no discernible difference in shape between  $|E(k, \omega, z)|^2$  at  $z = 0$ , where it is  $\propto 1/[1 + (\omega T_c)^2]$ , and at finite  $z$ , for small  $T_c$ .

If  $\tilde{\lambda}_{\max} \ll 1$ , i.e.,  $\tilde{I}_0 \lesssim 1$ , then the FSBS growth length,  $1/\lambda_{\max}$ , is large compared to the (vacuum)  $z$  correlation length,  $\propto k_0/k_m^2$ , and it is found, for small  $T_c$ , that a

quasiequilibrium is attained: Various low order statistical moments are roughly constant over the simulation range once  $k_m^2 z / k_0 \gtrsim 5$ , as seen in Fig. 3. A true equilibrium cannot be attained since  $\langle k^2 \rangle \equiv \langle |\nabla E|^2 \rangle / I_0$  grows due to scattering from density fluctuations as in Fig. 4. A dimensionless diffusion coefficient,  $\tilde{D} \equiv (k_0/k_m^4) \frac{d}{dz} \langle k^2 \rangle$  (proportional to the rate of angular diffusion), may be

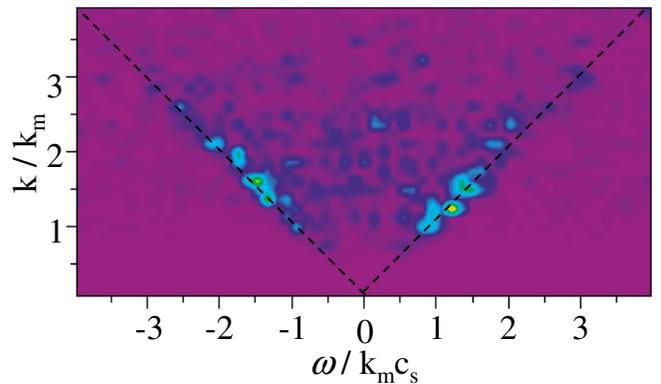


FIG. 1 (color). Density source power spectrum,  $k^4 |I(k, \omega)|^2$ , with  $\tilde{I}_0 \simeq 4.12$ ,  $\nu = 0.15$ ,  $\tilde{T}_c \simeq 0.033$ , and  $k_m^2 z / k_0 \simeq 7.9$ . The dashed lines are at  $\omega = \pm k c_s$ .

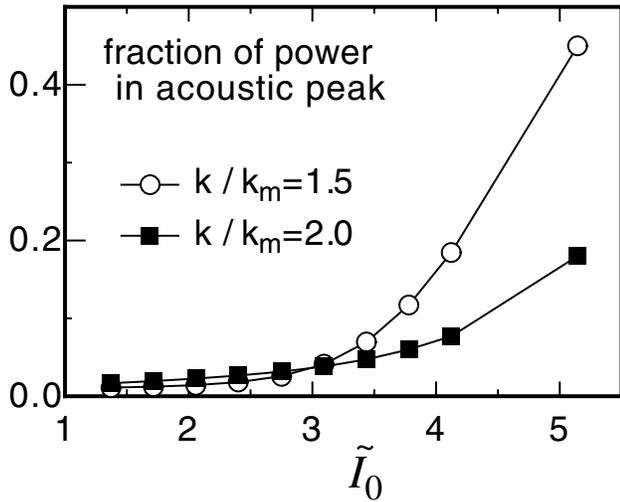


FIG. 2. Fractional power in acoustic peak of the *intensity fluctuation spectrum*, with parameters as in Fig. 1, except  $k_m^2 z/k_0 \approx 5.2$ . Note that the FSBS intensity threshold for  $k/k_m = 1.5(2.0)$  is about 3(4).

extracted from the data of Fig. 4 by fitting a smooth curve to  $\langle k^2 \rangle$  for  $5 < k_m^2 z/k_0 < 76$ , and evaluating its slope, extrapolated to  $z = 0$ . This yields a diffusion coefficient of  $4.4 \times 10^{-4}$ .

$\bar{D}$  may be compared to the solution of the stochastic Schroedinger equation (SSE) [15] with a self-consistent random potential [16],  $\rho$ , whose covariance,  $C^{\rho\rho}$  [ $C^{\rho\rho}$  is a quadratic functional of  $F(k)$ ] is evaluated as follows [17]. Take  $E$  as given by Eq. (3) with  $\rho$  set to 0 since it goes to zero with  $\bar{T}_c$ , and use it in Eq. (4), with  $\ln(1 + \rho) \rightarrow \rho$ , to evaluate  $C^{\rho\rho}$ . This is consistent only if  $\tilde{I}_0 < 1$ , so that the

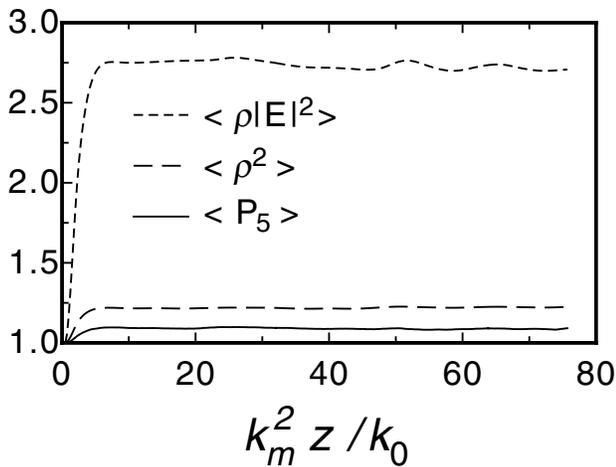


FIG. 3. A quasiequilibrium is attained with one point  $E$  fluctuations remaining nearly Gaussian, as evidenced by the small change in  $P_5$  [4], the fraction of power with intensity at least  $5I_0$ , but strongly modified  $I - \rho$  correlations. Parameters are  $\tilde{I}_0 \approx 0.53$ ,  $\nu = 0.3$ , and  $\bar{T}_c \approx 0.26$ . Each curve is normalized to its value at  $z = 0$ .

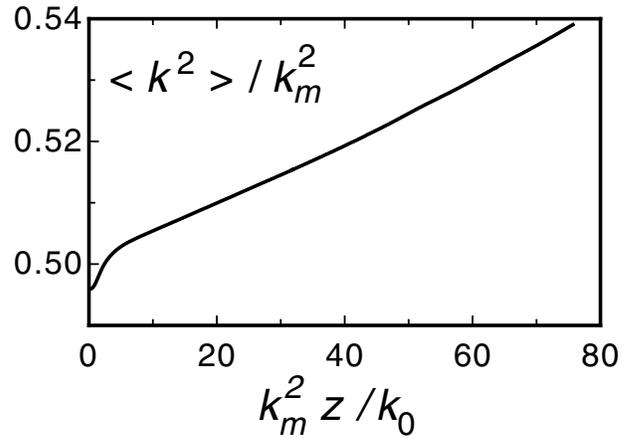


FIG. 4. For parameters of Fig. 3,  $\langle k^2 \rangle \equiv \langle |\nabla E|^2 \rangle / I_0$  increases little over the initial equilibration distance of roughly 5 in these units. The subsequent diffusion rate is  $4.4 \times 10^{-4}$ .

density response is stable except at small  $k/k_m$ . It follows, to leading order in  $\bar{T}_c$ , that the SSE prediction for  $\bar{D}$  for the top hat spectrum,

$$\bar{D}_{\text{SSE}} = \nu \bar{T}_c \tilde{I}_0^2 / 68.8 \dots, \quad (10)$$

has the value  $3.2 \times 10^{-4}$  for the parameters of Fig. 4. Note that  $\bar{D}_{\text{SSE}}$  is proportional to  $\langle \rho^2 \rangle$ , and the roughly 20% increase of  $\langle \rho^2 \rangle$  over its perturbative evaluation (see Fig. 3) used in the SSE accounts for about 1/2 of the difference between  $\bar{D}$  and  $\bar{D}_{\text{SSE}}$ .

We find that  $\bar{D}$  depends essentially on the spectral form,  $\langle |\hat{E}(k)|^2 \rangle = F(k)$ , e.g., for Gaussian  $F(k)$  with the same value of  $\langle k^2 \rangle$ ,  $D_{\text{Gaussian}} \approx 3D_{\text{top hat}}$ . A numerical example of this dependence is found in Figs. 4 and 5.  $\bar{D}$  changes by 40% over  $5 < k_m^2 z/k_0 < 76$ , because  $F(k)$  changes significantly as seen in Fig. 5. In this sense, for NIF relevant boundary conditions angular diffusion is an essential correction to the geometrical optics model, which (absent refraction) has constant  $F(k)$ .

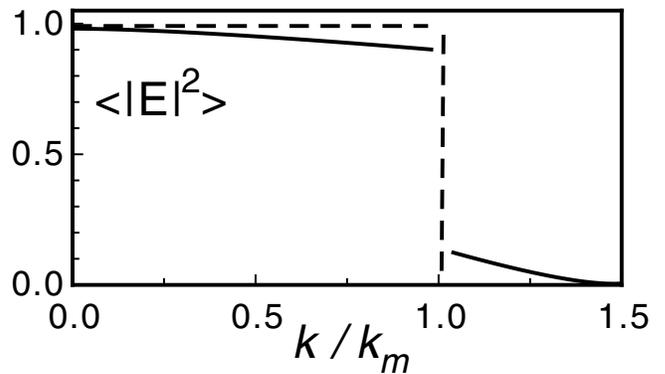


FIG. 5. Top hat boundary condition, dashed line, changes qualitatively over the propagation distance shown in Fig. 4: solid line at  $k_m^2 z/k_0 \approx 76$ .

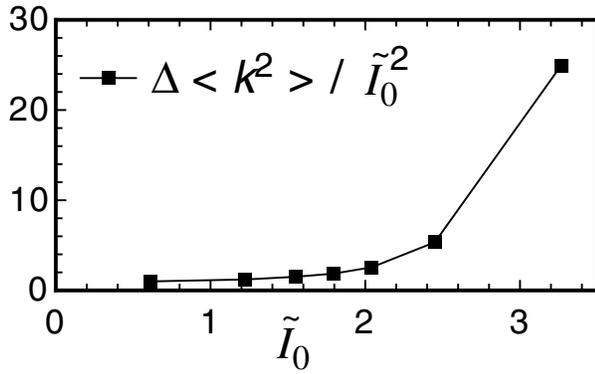


FIG. 6. Beam angular divergence rate increases rapidly with  $\tilde{I}_0$ .  $k_m z/k_0 \approx 15.7$ ,  $\nu = 0.0375$ , and  $\tilde{T}_c \approx 0.125$ . In contrast, Eq. (10) predicts a flat curve around 1.

Equation (10) implies that  $d/dz \langle (k/k_m)^2 \rangle \propto 1/k_m$ , while  $\lambda_{\max} \propto 1/k_m^8$ . If the diffusion length is smaller than the FSBS growth length, then propagation, which effectively increases  $k_m$ , will reinforce this ordering. This stability condition may be expressed as  $\tilde{D} > \tilde{\lambda}_{\max}$ , or qualitatively as [18]

$$\nu \tilde{T}_c > \tilde{I}_0^3. \quad (11)$$

This is a global condition, as opposed to the wave number dependent threshold,  $k_{\text{cutoff}}(\tilde{I}_0)$ . However, even if Eq. (11) is violated, it is not until  $k_{\text{cutoff}} \approx 1.5k_m$ , so that the peak of the density fluctuation spectrum is unstable, that FSBS has a strong effect. For these larger  $I_0$  values a quasi-equilibrium is not attained, and it is more useful to consider an integral measure,  $\Delta \langle k^2 \rangle(z) \equiv \langle k^2 \rangle(z) - \langle k^2 \rangle(0)$ , of the change in beam angular divergence rather than the differential measure  $D$ .  $\Delta/\tilde{I}_0^2$  is shown in Fig. 6 normalized to unity at  $\tilde{I}_0 = 0.61$ .

Note that we have not observed significant departure from Gaussian  $E$  fluctuations for  $\tilde{I}_0 < 2$  for the parameters of Fig. 6, which is consistent with the absence of self-focusing. Therefore in this regime the effect of FSBS is benign, and perhaps useful for NIF design purposes: Correlation lengths decrease at an accelerated pace compared to SSE for  $\tilde{I}_0 \sim 1$  with  $z$ , while electric field fluctuations stay nearly Gaussian. As a result [19], the intensity threshold for other instabilities (e.g., backscatter SBS) increases [20]. If  $\tilde{I}_0 > 4$ , there are large non-Gaussian fluctuations of  $E$ , which indicates strong self-focusing.

In conclusion, well above the FSBS threshold we observe strong self-focusing effects, while well below threshold beam propagation is diffusive in angle with essential corrections to geometric optics. In an intermediate range of intensities, the rate of angular diffusion increases with propagation. In the weak and intermediate regimes, the diffusion results in decreasing correlation lengths which could be beneficial for NIF.

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- [1] J. D. Lindl, *Phys. Plasmas* **2**, 3933 (1995).
- [2] It is also assumed that intensity fluctuations which self-focus on a time scale  $< T_c$  are not statistically significant. See H. A. Rose and D. F. DuBois, *Phys. Fluids B* **5**, 3337 (1993).
- [3] A. J. Schmitt and B. B. Afeyan, *Phys. Plasmas* **5**, 503 (1998).
- [4] C. H. Still *et al.*, *Phys. Plasmas* **7**, 2023 (2000).
- [5] R. H. Lehmburg and S. P. Obenschain, *Opt. Commun.* **46**, 27 (1983).
- [6] Fourier transform is in the  $xy$  plane with  $\mathbf{x} \rightarrow (x, y)$ ,  $\mathbf{k} \rightarrow (k_x, k_y)$ .
- [7] This requires that the speed of light  $c \gg L_z/T_c$ , where  $L_z$  is the  $z$  correlation length.
- [8] P. K. Kaw, G. Schmidt, and T. W. Wilcox, *Phys. Fluids* **16**, 1522 (1973).
- [9] A. J. Schmitt and R. S. B. Ong, *J. Appl. Phys.* **54**, 3003 (1983).
- [10] A. J. Schmitt, *Phys. Fluids* **31**, 3079 (1988).
- [11] H. A. Rose, D. F. DuBois, and D. Russell, *Sov. J. Plasma Phys.* **16**, 537 (1990).
- [12] D. N. Christodoulides, T. H. Coskun, M. Mitchell, and M. Segev, *Phys. Rev. Lett.* **78**, 646 (1997).
- [13] The precise condition depends on  $F_0$ : See Ref. [11] and H. A. Rose and D. F. DuBois, *Phys. Fluids B* **4**, 252 (1992). The first derivation of an analogous result for the case of the modulational instability of a broad Langmuir wave spectrum was done by A. A. Vedenov and L. I. Rudakov, *Sov. Phys. Dokl.* **9**, 1073 (1965); *Dokl. Akad. Nauk SSSR* **159**, 767 (1964).
- [14] If  $k$  is constrained by finite beam size effects to be, e.g.,  $k > 1/(\text{beam diameter})$ , then stability is regained for small enough  $I_0$ .
- [15] See, e.g., G. Bal, G. Papanicolaou, and L. Ryzhik, *Nonlinearity* **15**, 513 (2002).
- [16] This may be viewed as a special case of the wave kinetic equation [see, e.g., V. E. Zakharov, V. S. Lvov, and G. Falkovich, *Kolmogorov Spectra of Turbulence I: Wave Turbulence* (Springer-Verlag, New York, 1992)].
- [17] It is assumed that the density fluctuations are due only to the beam itself, in contrast to the experimental configuration found in J. D. Moody *et al.*, *Phys. Plasmas* **7**, 2114 (2000).
- [18] If collisional absorption is included in Eq. (3) with rate  $\kappa$ , then for  $T_c \rightarrow 0$  the stability condition is  $\lambda_{\max} < \kappa$ .
- [19] B. Afeyan (private communication) has reached somewhat similar conclusions in the context of self-focusing.
- [20] H. A. Rose and D. F. DuBois, *Phys. Rev. Lett.* **72**, 2883 (1994).