Problem 1, hw 09. Math 462/512.

Consider solution of the stationary Schrödinger equation

$$\psi_{xx} + \frac{2m}{\hbar^2}[E - V(x)]\psi = 0, \tag{1}$$

where $E > 0$ is the energy (constant) and m, $\hbar$ are the constants. The potential $V(x)$ is periodic with the period $a > 0$ such that $V(x) = 0$ for $-\frac{1}{2}(a-b) < x < \frac{1}{2}(a-b)$ and $V(x) = -U_0 < 0$ for $-\frac{1}{2}(a+b) < x < -\frac{1}{2}(a-b)$, where $a > b > 0$ and U_0 is the constant. Consider Floquet solution $\psi(x) = e^{ikx}y(x)$ of the stationary Schrödinger equation such that $y(x) = y(x+a)$. Find expression for k.
Hint: Use that ψ and ψ_x are continuous functions for $-\infty < x < \infty$.