EXERCISES

1. Sketch the following sets and determine which are domains:

(a) $|z - 2 + i| \leq 1$; *(b)* $|2z + 3| > 4$;
(c) $\text{Im}\, z > 1$; *(d)* $\text{Im}\, z = 1$;
(e) $0 \leq \arg z \leq \pi/4 \ (z \neq 0)$; *(f)* $|z - 4| \geq |z|$.

Ans. *(b)*, *(c)* are domains.

2. Which sets in Exercise 1 are neither open nor closed?

Ans. *(e)*.

3. Which sets in Exercise 1 are bounded?

Ans. *(a)*.

4. In each case, sketch the closure of the set:

(a) $-\pi < \arg z < \pi \ (z \neq 0)$; *(b)* $|\text{Re}\, z| < |z|$;
(c) $\text{Re}\left(\dfrac{1}{z}\right) \leq \dfrac{1}{2}$; *(d)* $\text{Re}(z^2) > 0$.

5. Let S be the open set consisting of all points z such that $|z| < 1$ or $|z - 2| < 1$. State why S is not connected.

6. Show that a set S is open if and only if each point in S is an interior point.

7. Determine the accumulation points of each of the following sets:

(a) $z_n = i^n \ (n = 1, 2, \ldots)$; *(b)* $z_n = i^n/n \ (n = 1, 2, \ldots)$;
(c) $0 \leq \arg z < \pi/2 \ (z \neq 0)$; *(d)* $z_n = (-1)^n (1 + i)\dfrac{n-1}{n} \ (n = 1, 2, \ldots)$.

Ans. *(a)* None; *(b)* 0; *(d)* $\pm(1 + i)$.

8. Prove that if a set contains each of its accumulation points, then it must be a closed set.

9. Show that any point z_0 of a domain is an accumulation point of that domain.

10. Prove that a finite set of points $z_1, z_2, \ldots, z_n$ cannot have any accumulation points.