EXERCISES

1. Reduce each of these quantities to a real number:

$$(a)\ \frac{1+2i}{3-4i}+\frac{2-i}{5i};\qquad (b)\ \frac{5i}{(1-i)(2-i)(3-i)};\qquad (c)\ (1-i)^4.$$

Ans. *(a)* $-2/5$; *(b)* $-1/2$; *(c)* -4.

2. Show that

$$\frac{1}{1/z}=z \qquad (z\neq 0).$$

3. Use the associative and commutative laws for multiplication to show that

$$(z_1z_2)(z_3z_4)=(z_1z_3)(z_2z_4).$$

4. Prove that if $z_1z_2z_3=0$, then at least one of the three factors is zero.

Suggestion: Write $(z_1z_2)z_3=0$ and use a similar result (Sec. 3) involving two factors.

5. Derive expression (6), Sec. 3, for the quotient z_1/z_2 by the method described just after it.

6. With the aid of relations (10) and (11) in Sec. 3, derive the identity

$$\left(\frac{z_1}{z_3}\right)\left(\frac{z_2}{z_4}\right)=\frac{z_1z_2}{z_3z_4} \qquad (z_3\neq 0,\ z_4\neq 0).$$

7. Use the identity obtained in Exercise 6 to derive the cancellation law

$$\frac{z_1z}{z_2z}=\frac{z_1}{z_2} \qquad (z_2\neq 0,\ z\neq 0).$$

8. Use mathematical induction to verify the binomial formula (13) in Sec. 3. More precisely, note that the formula is true when $n=1$. Then, assuming that it is valid when $n=m$ where m denotes any positive integer, show that it must hold when $n=m+1$.

Suggestion: When $n=m+1$, write

$$(z_1+z_2)^{m+1}=(z_1+z_2)(z_1+z_2)^m=(z_2+z_1)\sum_{k=0}^{m}\binom{m}{k}z_1^kz_2^{m-k}$$
$$=\sum_{k=0}^{m}\binom{m}{k}z_1^kz_2^{m+1-k}+\sum_{k=0}^{m}\binom{m}{k}z_1^{k+1}z_2^{m-k}$$

and replace k by $k-1$ in the last sum here to obtain

$$(z_1+z_2)^{m+1}=z_2^{m+1}+\sum_{k=1}^{m}\left[\binom{m}{k}+\binom{m}{k-1}\right]z_1^kz_2^{m+1-k}+z_1^{m+1}.$$