Spring 2017. MATH 579 - 001: Nonlinear Partial Differential Equations with Applications to Optics and Waves

Instructor: Pavel Lushnikov

TR 12:30-13:45 pm. Room : Dane Smith Hall 334

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Course web page:

math.unm.edu/∼plushnik/teaching/math579nonlinearwaves2017spring/math579NonlinearWaves2017Springsyllabus.pdf

We surrounded by waves in our everyday life such as light, sound and oceanic waves. Linear waves are described by linear partial differential equations (PDE). Waves become nonlinear when we can not consider their amplitudes being arbitrary small. Contrary to linear waves, nonlinear waves are described by nonlinear PDE, they interact with each other and often form new structures. This course will be focused on modern theory of nonlinear waves with application to nonlinear optics, Bose-Einstein condensation, water waves, optical fiber communications, electrical engineering and hydrodynamics. The introduction to the Hamiltonian formulation of nonlinear waves will be given. We will derive universal models of nonlinear waves including Burgers equation, Korteweg-de Vries equation, nonlinear Schrödinger equation and Kadomtsev-Petviashvili equation. Brief introduction to the spectral method for numerical solution of partial differential equations will be also given. The course is intended for mathematicians, physicists, computer scientists and engineers. Topics covered will include:

1. Linear waves in optics and hydrodynamics. Phase and group velocity.
2. Nonlinear interaction of waves. Nonlinear Schrödinger equation.
3. Introduction to the inverse scattering transform. Integrability of nonlinear Schrödinger equation.
4. Nonlinear light waves. Nonlinear optics. Kerr nonlinearity.
5. Optical communications and solitons.
6. Nonlinear waves in more than one dimension. Kadomtsev-Petviashvili equation. Nonlinear Schrödinger equation in dimension two. Wave collapse and formation of singularities.
7. Vibrating nonlinear string. Fermi-Pasta-Ulam problem. Multiscale expansion.
8. Basic equations of hydrodynamics. Euler equations. Navier-Stokes equations.
9. Numerical solution of partial differential equations. Fast Fourier Transform. Spectral and pseudospectral methods.

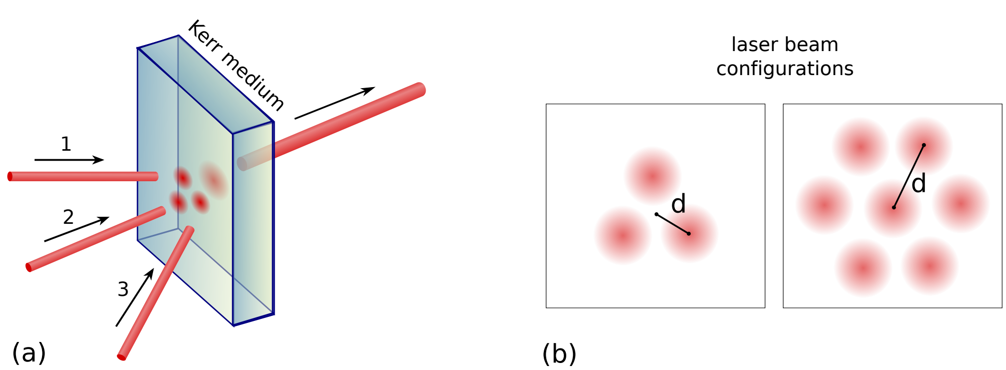

Figure 1: A schematic of the nonlinear beam combining through collapse. (a) An array of beams with non-correlated phases enters a nonlinear Kerr medium and produce diffraction-limited beam through the self-focusing (collapse) of multiple laser beams into the single output beam. (b) Cross-section of these beams before entering medium for 3 and 7 beams.

Figure 2: Breaking of water waves.

Optional reading

References

[1] Eryk Infeld and George Rowlands, Nonlinear Waves, Solitons and Chaos, Cambridge University Press, 2000 ISBN-10: 0521635578

[2] Alan C. Newell, Solitons in mathematics and physics, Philadelphia, Pa. : Society for Industrial and Applied Mathematics, 1987 ISBN-10: 0898711967

[3] Jerome V. Moloney and Alan C. Newell, Nonlinear Optics, Westview Press, 2004 ISBN-10: 0813341183

[4] S. Novikov, S.V. Manakov, L.P. Pitaevskii and V.E. Zakharov, Theory of Solitons: The Inverse Scattering Method, 1984 ISBN-10: 0306109778