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A Robot Manipulator With 16 Real Inverse Kinematic Solution Sets

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Abstract

A solution search algorithm based on a one-dimensional numerical approach to the inverse kinematic problem (presented in an earlier paper) led to the discovery of a six-DOF manipulator able to position and orient its end-effector in 16 distinct configurations for a given end-effector pose (position and orientation). This paper discusses the consequences of such a discovery and presents a description of the manipulator, the end-effector pose, and the 16 kinematic solutions.

1. Introduction

The inverse kinematic problem is at the center of computer control algorithms for robot manipulators. To be able to position and orient the end-effector in a given fashion, a corresponding set of joint-variable values must be computed. The complexity of this problem is highly dependent on the geometry of the manipulator, and the inverse kinematic solution is, in general, not unique. The exact number of solutions depends on the manipulator architecture as well as the desired end-effector pose.

The problem of finding all possible solutions has been addressed by Tsai and Morgan (1984), who applied homotopy map techniques to the inverse kinematic problem of 6- and 5-revolute-degree-of-freedom arms and developed a numerical method guaranteed to find all isolated solutions. An application example of this homotopy map method proved that the number of real inverse kinematic solutions could be as high as 12.

Duffy and Crane (1980) have shown that the inverse kinematic problem of 6-DOF arms can be expressed in terms of a 32nd-degree polynomial in one joint variable. Some more recent work by Lee and Liang (1988) reduces the degree of the polynomial to 16. This result puts a bound of 16 on the number of distinct configurations by which a 6-DOF general manipulator can realize a given end-effector pose. In this paper, we present the first manipulator to actually achieve this maximum number of solutions, thereby closing the proof that 16 is indeed the maximum number of inverse kinematic solutions for 6-DOF manipulators.

2. The Inverse Kinematics Problem

A robot arm can be modeled as a kinematic chain with a fixed link, called the base, and a free link at the other end of the chain, referred to as the end-effector. The links along the chain are numbered from 0 to n for an n-link, n-DOF manipulator. According to the Denavit and Hartenberg (1955) notation, a frame of reference F_i assigned to link i has a position and orientation fully described by the four parameters d_i, θ_i, a_i, and α_i with respect to the preceding frame F_{i-1}. Figure 1 shows a manipulator with the assigned frames.

A vector $\mathbf{v}^i = [v_x^i, v_y^i, v_z^i, 1]^T$, given with respect to frame F_i, can be related to its expression $\mathbf{v}^{i-1}$ in frame F_{i-1} by use of the homogeneous matrix transform $\mathbf{A}_i$ and its inverse $\mathbf{A}_i^{-1}$ (Paul 1981),

$$\mathbf{v}^{i-1} = \mathbf{A}_i \mathbf{v}^i \quad \text{and} \quad \mathbf{v}^i = (\mathbf{A}_i^{-1})\mathbf{v}^{i-1}$$

with

$$\mathbf{A}_i = \begin{bmatrix} C_i & -S_i\tau_i & S_i\sigma_i & a_iC_i \\ S_i & C_i\tau_i & -C_i\sigma_i & a_iS_i \\ 0 & \sigma_i & \tau_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} & \mathbf{R}_i & & \mathbf{l}_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (1)$$

where $C_i = \cos(\theta_i)$, $S_i = \sin(\theta_i)$, $\tau_i = \cos(\alpha_i)$, and $\sigma_i = \sin(\alpha_i)$.

The rotation matrix $\mathbf{R}_i$ necessary to align the unit vectors of F_i with those of F_{i-1} is the upper left 3×3 matrix in $\mathbf{A}_i$ and vector $\mathbf{l}_i = [a_iC_i, a_iS_i, d_i]^T$ positions the origin of F_i with respect to F_{i-1}.

Most existing industrial arms are designed so that all the twist angles α_i along the manipulator are either 0 or $\pi/2$. Such manipulators are said to be orthogonal (Doty 1986; Manseur and Doty 1988).

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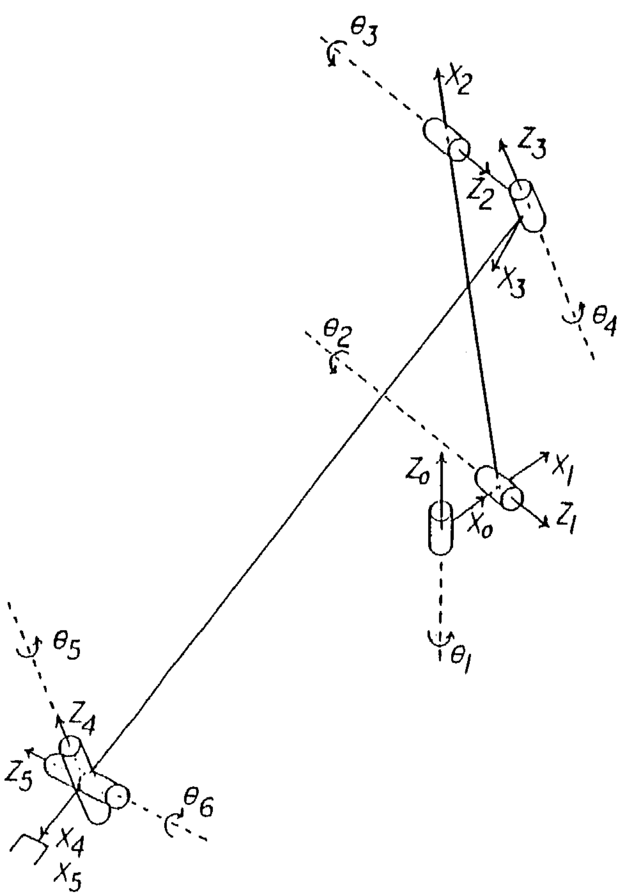
Fig. 1. OM25 manipulator structure with link-frames.



If the position and orientation of the end-effector frame are expressed by a pose matrix of the form

$$\mathbf{P} = \begin{bmatrix} n_x & b_x & t_x & p_x \\ n_y & b_y & t_y & p_y \\ n_z & b_z & t_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{n} & \mathbf{b} & \mathbf{t} & \mathbf{p} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} & \mathbf{R} & & \mathbf{p} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

with respect to the base frame $\mathbf{F}_0$, then the inverse kinematics problem consists of finding the sets of joint-variable values (solution sets) that will satisfy the matrix equation

$$\mathbf{A}_1 \cdot \mathbf{A}_2 \cdot \mathbf{A}_3 \cdot \mathbf{A}_4 \cdot \mathbf{A}_5 \ldots \mathbf{A}_n = \mathbf{P}. \quad (3)$$

For 6-DOF manipulators, n is equal to 6, and equation (3) can have several solutions. In general, the exact number of real solutions is bounded by 16 (Lee and Liang 1988), and the largest number actually reported in the literature was 12 (Tsai and Morgan, 1984), but for some manipulator configurations, two or more joint axes align, and the manipulator can then achieve the corresponding pose in infinitely many ways. Such a condition, where the manipulator loses degrees of freedom, is called a degeneracy and forces the manipulator Jacobian to become singular (Craig 1986).

In the next section we present an orthogonal manipulator able to achieve a given end-effector pose in 16 distinct configurations. To verify that the manipulator is non-degenerate at each of those 16 configurations, we compute the determinant of the manipulator Jacobian.

Table 1. OM25 Orthogonal Manipulator Kinematic Parameters

i	d_i	θ_i	a_i	α_i
1	0	θ_1	a_1	$\pi/2$
2	0	θ_2	a_2	0
3	d_3	θ_3	0	$\pi/2$
4	0	θ_4	a_4	0
5	0	θ_5	0	$\pi/2$
6	0	θ_6	0	0

3. A Manipulator with 16 Solution Sets

Figure 1 shows an orthogonal manipulator (referred to as the OM25 manipulator) whose kinematic parameters are given in Table 1 with $d_3 = 0.2$, $a_1 = 0.3$, $a_2 = 1$, and $a_4 = 1.5$.

This manipulator can realize the end-effector pose

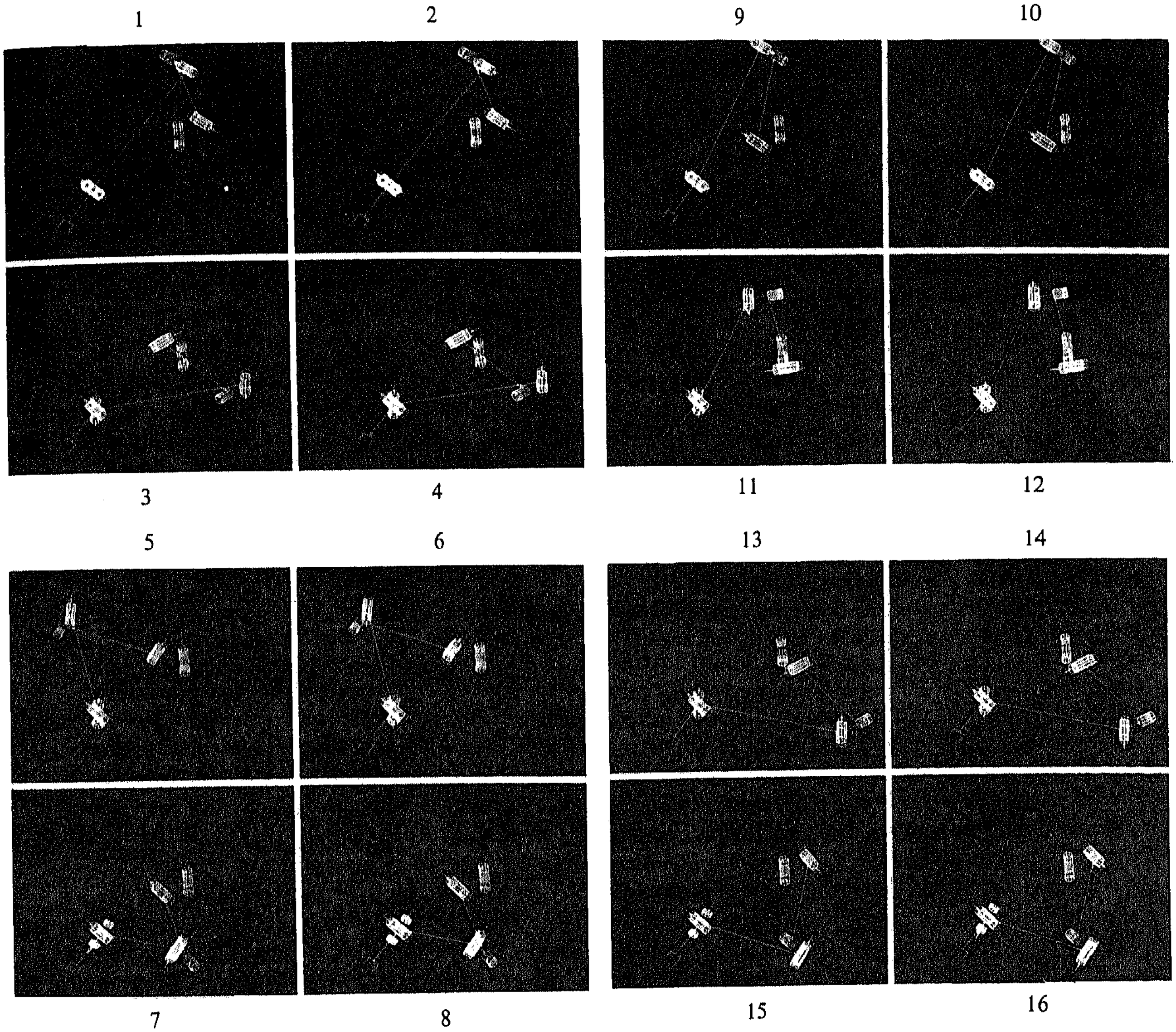
$$P = \begin{bmatrix} -0.760117 & -0.641689 & 0.102262 & -1.140175 \\ 0.133333 & 0 & 0.991071 & 0 \\ -0.635959 & 0.766965 & 0.085558 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

in 16 different ways.

Equation (3), with the parameters given in Table 1 and pose (4), was solved by use of a fast inverse kinematic procedure fully described in Manseur and Doty (1988). The 16 solution sets found are listed in Table 2. This result is of importance since it provides the first tangible proof that a 6-DOF manipulator can actually achieve a given pose with 16 distinct configurations. It is also interesting to note that this large number of solutions can be realized by an orthogonal manipulator with a fairly simple geometry.

The symbolic manipulator Jacobian has its simplest expression in frame 3 (midframe) for a 6-DOF manipulator

Fig. 2. Computer graphics simulation of the OM25 robot in the 16 configurations of Table 2.



(Renaud, 1980 a,b; Doty, 1987). For the robot described here, the midframe Jacobian $\mathbf{J}^3$, computed using tables given in Doty (1987), is

$$\mathbf{J}^3 = \begin{bmatrix} 0.2\,C_{23} & S_3 & 0 & 0 & 1.5\,S_4 & 0 \\ -0.3 - C_2 & 0 & 0 & 0 & -1.5\,C_4 & 0 \\ 0.2\,S_{23} & -C_3 & 0 & 0 & 0 & -1.5\,C_5 \\ S_{23} & 0 & 0 & 0 & 0 & S_{45} \\ 0 & 1 & 1 & 0 & 0 & -C_{45} \\ -C_{23} & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \quad (5)$$

where C_{ij} and S_{ij} stand for the cosine and the sine, respectively, of $(\theta_i + \theta_j)$. The determinant of the manipulator Jacobian is independent of the frame of expression and can be easily obtained from matrix (5):

$$\det(\mathbf{J}^3) = 1.5\{C_3 S_{45}[S_4(0.3 + C_2) - 0.2\,C_{23}C_4] - S_3 S_{23} C_4(1.5\,C_5 + 0.2\,S_{45})\}. \quad (6)$$

The values of det $(\mathbf{J}^3)$, listed in Table 2, prove that all 16 solutions found correspond to non-degenerate configurations of the OM25 robot arm.

Figure 2 shows photographs of a computer graphics simulation of the OM25 manipulator in the 16 configurations

Table 2. Solution Sets for Pose (4)

n	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	det (J^3)
1	0.000	107.458	112.460	−7.662	0.000	0.000	1.310
2	0.000	107.458	−67.540	−172.338	180.000	180.000	1.310
3	88.670	−176.682	−178.394	−63.284	157.829	139.944	−0.800
4	88.670	−176.682	1.606	−116.716	22.171	−40.056	−0.800
5	113.841	4.741	−179.093	−55.954	−63.659	−42.463	−1.256
6	113.841	4.741	0.907	−124.046	−116.341	137.537	−1.256
7	168.703	−104.205	146.556	−16.393	−170.903	98.216	0.803
8	168.703	−104.205	−33.444	−163.607	−9.097	−81.784	0.803
9	180.000	107.458	−147.375	−7.662	−164.675	180.000	0.732
10	180.000	107.458	32.625	−172.338	−15.325	0.000	0.732
11	−120.748	173.066	−178.472	31.328	−146.087	142.605	−0.717
12	−120.748	173.066	1.528	148.672	−33.913	−37.395	−0.717
13	−96.292	−5.766	−179.142	38.477	51.922	−39.631	−1.441
14	−96.292	−5.766	0.858	141.523	128.078	140.369	−1.441
15	−11.768	−105.495	−114.490	1.243	6.408	−79.398	1.318
16	−11.768	−105.495	65.510	178.757	173.592	100.602	1.318

listed in Table 1. Figure 1 is a hand drawing of this manipulator in configuration 1 (i.e., corresponding to solution 1), with all link frames clearly indicated, and to help differentiate between solutions with common values of θ_1 and θ_2, we have attempted to indicate the direction of axis vectors $\mathbf{z}_1$, $\mathbf{z}_3$, $\mathbf{z}_4$, and $\mathbf{z}_5$ on the photographs. The position and orientation of the end-effector and the base frame (as shown on Figure 1) are the same for all 16 representations of Figure 2.

4. Conclusion

The difficulty of finding a manipulator with 16 solution sets for a given pose stems from the fact that the number of inverse kinematic solutions depends not only on the manipulator kinematic structure parameters, but also on the end-effector pose. The problem then was one of finding a candidate manipulator that would support 16 different configurations for a pose that also had to be determined.

A fast inverse kinematic technique (Manseur and Doty 1988) and the geometric insight provided by numerous inverse kinematic computations led to the discovery of the manipulator in Figure 1 and an end-effector pose yielding 16 inverse kinematic solution sets. Once an end-effector pose can be achieved by 16 configurations, it is interesting to note that other such poses may be found easily by substituting an arbitrary value of θ_1 in any of the solution sets of Table 2. The resulting end-effector pose still yields 16 solution sets.

The work of Lee and Liang (1988) puts an upper bound of 16 on the number of inverse kinematic solutions of a general 6-DOF manipulator. The manipulator of Figure 1, therefore, establishes 16 as the least upper bound on the number of inverse kinematic solution sets for 6-DOF arms.

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References

Craig, J. J. 1986. *Introduction to Robotics Mechanics and Control.* Addison-Wesley.

Denavit, J., and Hartenberg, R. S. 1955. A kinematic notation for lower-pair mechanisms based upon matrices. *J. App. Mech.* 77:215–221.

Doty, K. L. 1987. Tabulation of the symbolic midframe jacobian of a robot manipulator. *Int. J. Robotics Res.* 6(4):85–97.

Doty, K. L. 1986. Machine intelligence laboratory report MIL(10.86.1). Elec. Eng. Dept., U. of Florida. Gainesville, Fl.

Duffy, J., and Crane, C. 1980. A displacement analysis of the general spatial 7-link, 7-R mechanisms. *Mechanisms and Machine Theory* 15(3):153–159.

Lee H. Y., and Liang C. G. 1988. Displacement analysis of the general spatial 7-link 7-R mechanism. *Mechanisms and Machine Theory* 23(3):219–226.

Manseur, R., and Doty, K. L. 1988. A fast algorithm for inverse kinematic analysis of robot manipulators. *Int. J. Robotics Res.* 7(3):52–63.

Paul, R. P. 1981. *Robot manipulators: Mathematics, programming, and control.* Cambridge: MIT Press.

Renaud, M. 1980a. Calcul de la matrice Jacobienne nécéssaire à la commande coordonnée d'un manipulateur. *Mechanisms and Machine Theory* 15(2):81–91.

Renaud, M. 1980b. Contribution à la modelisation et à la commande dynamique des robots manipulateurs. Thèse de Docteur d'Etat. Univ. Paul Sabatier de Toulouse (Sciences).

Tsai, L. W., and Morgan, A. P. 1984. Solving the kinematics of the most general six- and five-degree-of-freedom manipulators by continuation methods. ASME Paper 84-DET-20. ASME Design Engineering Technical Conference. Cambridge, Mass.

Reuleaux Pairs and Surfaces That Cannot Be Gripped

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Abstract

It is well known that there exist surfaces whose motion cannot be completely constrained by non-frictional contact forces. We give a new proof of the classification of these surfaces based on group theory. Having derived a simple characterization of these "surfaces that cannot be gripped," we show that they are equivalent to the Reuleaux lower pairs. The proof emphasizes the symmetry of the surfaces rather than their analytic form. We also show that the screw system of such a surface is isomorphic to the Lie algebra of the surface's symmetry group.

1. Introduction

The problem of constraining the motion of a rigid body by applying surface contact forces has a long history. Reuleaux himself (1875) studied this "gripper" problem. Recently many workers have observed that there exist surfaces that cannot be completely constrained by any number of frictionless surface contacts. All the examples provided so far are surfaces of the Reuleaux lower pairs. But is this always the case? Do there exist surfaces that cannot be gripped in the above sense but are not Reuleaux lower pairs? Intuitively the answer is no, but there seems to be no proof of this in the literature.

The proof we present here leads naturally to a classification of the Reuleaux lower pairs. The traditional classifica-