A reduced-order partial differential equation model for dynamics of the flow in a thermosyphon

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Flow in a closed loop thermosyphon heated from below exhibits a sequence of bifurcations with increasing Grashof number. Using the Navier-Stokes equations in the Boussinesq approximation we have derived a model where, in the case of a slender circular loop, the first Fourier modes exactly decouple from all other Fourier modes, leaving a system of three coupled nonlinear partial differential equations that completely describes the flow in the thermosyphon. We have characterized the flow through two bifurcations, identifying stable periodic solutions for flows of Prandtl number greater than 18.5, a much lower value than predicted previously. Because of the quadratic nonlinearity in this system of equations, it is possible to find the global stability limit, and we have proved it is identical to the first bifurcation point.

The numerical study of the model equations is based on a highly accurate Fourier-Chebyshev spectral method, combined with asymptotic analysis at the various bifurcation points. Three-dimensional computations with a finite element method computational fluid dynamics code (MPSalsa), are also pursued. All three approaches are in close agreement.