Show your work if you wish to receive credit.

Let X be a random variable with p.m.f. \( f(x) = \frac{8}{7} \left(\frac{1}{2}\right)^x \), \( x = 1, 2, 3 \)

1. a) Find \( P(X \leq 1) \)

This is problem 3.16. Solution posted for all parts.

b) \( P(X > 1) \)

c) \( P(2 < X < 6) \)

2. Find the c.d.f. of X. You must define it over the entire real line and sketch the graph.

\[
F(x) = \begin{cases} 
0, & x < 1 \\
\frac{4}{7}, & 1 \leq x < 2 \\
\frac{6}{7}, & 2 \leq x < 3 \\
1, & 3 \leq x 
\end{cases}
\]

3. Find the mean and variance of X

\[
\mu = E(X) = \sum_{all\, x} xf(x) = 1(4/7) + 2(2/7) + 3(1/7) = \frac{11}{7}
\]

\[
E(X^2) = \sum_{all\, x} x^2 f(x) = 1^2(4/7) + 2^2(2/7) + 3^2(1/7) = \frac{21}{7} = 3
\]

\[
\sigma^2 = E(X^2) - \mu^2 = 3 - \left(\frac{11}{7}\right)^2 = \frac{147 - 121}{49} = \frac{26}{49}
\]
4. An inspector working for a manufacturing company has a 99% chance of correctly identifying defective items and a 0.5% chance of incorrectly classifying a good item as defective. The company has evidence that its line produces 0.9% of nonconforming items.

(a) What is the probability that an item selected for inspection is classified as defective?

This is problem 2.98. The solution is posted.

(b) If an item selected at random is classified as nondefective, what is the probability that it is indeed good?