1. Samples of 20 parts from a metal finishing process are selected every hour. Typically 1% of the parts require rework. Let X denote the number of parts in the sample of 20 that require rework. A process problem is suspected if X exceeds its mean by more than three standard deviations.

a) If the percentage of parts that require rework remains at 1%, what is the probability that X exceeds its mean by more than three standard deviations?

\[ X \sim Bin(n = 20, p = .01) \]
\[ \mu = E(X) = 20(.01) = .2, \quad \sigma = \sqrt{20(.01)(.99)} = .445 \]
\[ \mu + 3\sigma = .2 + 1.335 = 1.535 \]
\[ P(X > \mu + 3\sigma) = P(X > 1.535) = P(X > 1) = 1 - P(X \leq 1) \]
\[ = 1 - [(.99)^{20} + 20(.01)(.99)^{19}] = 1 - [0.8179 + 0.1652] \]
\[ = 1 - 0.9831 = 0.0169 \]

b) If the rework percentage increases to 4%, what is the probability that X exceeds 1?

\[ X \sim Bin(n = 20, p = .04) \]
\[ P(X > 1) = 1 - P(X \leq 1) \]
\[ = 1 - [(,.96)^{20} + 20(.04)(.96)^{19}] = 1 - [0.4420 + 0.3683] \]
\[ = 1 - 0.8103 = 0.1897 \]

2. Suppose the random variable X has a geometric distribution with a mean of 2.5. Calculate P(X > 3).

\[ \mu = \frac{1}{p}, \text{ so } p = \frac{1}{\mu} \]
\[ \mu = 2.5 \Rightarrow p = \frac{1}{2.5} = 0.4 \]
\[ P(X > 3) = 1 - P(X \leq 3) = 1 - [P(X = 1) + P(X = 2) + P(X = 3)] \]
\[ = 1 - [(,.6)^0(.4) + (.6)^1(.4) + (.6)^2(.4)] = 1 - (.4)(1 - (.6)^3)/1 - .6 = (.6)^3 = .216 \]

3. Return to the setup of problem 1. If the rework percentage is at 1%, how many hours do I expect to collect samples until I finally see one that needs rework?

Each hour I collect 20, so my probability of seeing at least one each hour is
\[ p = 1 - (.99)^{20} = 1 - .8179 = 0.1821. \] If \( Y = \) hours to see first, then \( Y \sim Geometric(p = 0.1821). \)
\[ E(Y) = \frac{1}{0.1821} = 5.49 \text{ hours} \]