Stat 345 Solutions - Section 2.6

Problem 2-84

\[ P(A \cap B) = \frac{80}{100}, P(A) = \frac{82}{100}, P(B) = \frac{80}{100} \]

Since \( P(A \cap B) \neq P(A)P(B) \) then A and B are not independent.

Problem 2-87

Let \( H_i \) denote the event that the \( i^{th} \) specimen contains high levels of contamination.

(a) \( P(H'_1 \cap H'_2 \cap H'_3 \cap H'_4 \cap H'_5) = P(H'_1)P(H'_2)P(H'_3)P(H'_4)P(H'_5) \) by independence.
\[ P(H'_i) = 0.9, \text{ so the answer is } (0.9)^5 = 0.590. \]

(b) \( A_1 = H'_1 \cap H'_2 \cap H'_3 \cap H'_4 \cap H'_5 \)
\[ A_2 = H'_1 \cap H'_2 \cap H'_3 \cap H'_4 \cap H_5 \]
\[ A_3 = H'_1 \cap H'_2 \cap H'_3 \cap H'_4 \cap H_5 \]
\[ A_4 = H'_1 \cap H_2 \cap H'_3 \cap H'_4 \cap H_5 \]
\[ A_5 = H'_1 \cap H_2 \cap H_3 \cap H'_4 \cap H'_5 \]

We want \( P(A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5) \). By independence \( P(A_i) = (0.9)^4(0.1) = .0656 \) for each of the \( A_i \). The \( A_i \) also are mutually exclusive, so
\[ P(A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5) = \sum_{i=1}^{5} P(A_i) = 5(.0656) = 0.328. \]

(c) If B is the event in part (a), i.e. B is the event no sample contains high levels of contamination, then \( P(B) = .059 \). We want \( P(B') = 1 - P(B) = 1 - 0.59 = 0.41 \).

Problem 2-90

Let \( A \) = event all upper devices function, \( B \) = event all lower devices function.
Everything operates independently, so
\[ P(A) = (0.9)(0.8)(0.7) = 0.504, \quad P(B) = (0.95)(0.95)(0.95) = 0.8574, \quad \text{and} \]
\[ P(A \cap B) = (0.504)(0.8574) = 0.4321. \]
The circuit operates is the event \( A \cup B \) and \( P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.504 + 0.8574 - 0.4321 = 0.9293 \).