Stat 345 Solutions - Section 4.2

Problem 4-1

(a) \( P(X > 1) = \int_1^\infty e^{-x}dx = -e^{-x}\bigg|_1^\infty = 0 + e^{-1} = 0.3679 \)
(b) \( P(1 < X < 2.5) = \int_1^{2.5} e^{-x}dx = -e^{-x}\bigg|_1^{2.5} = -0.0821 + 0.3679 = 0.2858 \)
(c) \( P(X = 3) = 0 \)
(d) \( P(X < 4) = \int_0^4 e^{-x}dx = -e^{-x}\bigg|_0^4 = -0.0183 + 1 = 0.9817 \)
(e) \( P(X \geq 3) = \int_3^\infty e^{-x}dx = -e^{-x}\bigg|_3^\infty = 0 + 0.0498 = 0.0498 \)

Problem 4-2

(a) We want to find \( x \) such that \( P(X > x) = 0.10 \).
\[
P(X > x) = \int_x^\infty e^{-x}dx = -e^{-x}\bigg|_x^\infty = 0 + e^{-x}.
\]
Thus, we have \( e^{-x} = 0.1 \) and so \( x = 2.3 \).

(b) We want to find \( x \) such that \( P(X \leq x) = 0.10 \).
\[
P(X \leq x) = \int_0^x e^x dx = -e^{-x}\bigg|_0^x = -e^{-x} + 1.
\]
Thus, we have \( 1 - e^{-x} = 0.1 \) and so \( e^{-x} = 0.9 \), which gives \( x = 0.1054 \).

Problem 4-5

(a) \( P(X > 0) = \int_1^0 1.5x^2dx = 1.5\frac{x^3}{3}\bigg|_0^1 = \frac{3}{2}(1) = 0.5 \). Alternatively, you can just notice that the density is symmetric around 0, and so the probability of being greater than 0 is \( \frac{1}{2} \).

(b) \( P(X > 0.5) = \int_0^{1/2} 1.5x^2dx = 1.5\frac{x^3}{3}\bigg|_0^{1/2} = \frac{1}{2}(1 - \frac{1}{8}) = 0.4375 \)

(c) \( P(-0.5 \leq X \leq 0.5) = \int_{-0.5}^{0.5} 1.5x^2dx = 1.5\frac{x^3}{3}\bigg|_{-0.5}^{0.5} = \frac{1}{2}(\frac{1}{8} + \frac{1}{8}) = \frac{1}{8} = 0.125 \)

(d) \( P(X < -2) = 0 \)

(e) \( P(X < 0 \text{ or } X > -0.5) = 1 \), since this includes all values between -1 and 1. Alternatively, \( P(X < 0 \text{ or } X > -0.5) = P(X < 0) + P(X > -0.5) - P(X < 0 \text{ and } X > -0.5) = 0.5 + \int_{-0.5}^{-0.5} 1.5x^2dx - \int_{-0.5}^{0} 1.5x^2dx = 0.5 + 0.5 - \frac{0.5^3}{2} - (0 - \frac{0.5^3}{2}) = 1 \)

(f) \( P(x < X) = \int_x^1 1.5w^2dw = 0.5w^3\bigg|_x^1 = 5(1 - x^3), -1 < x < 1 \), so \( P(x < X) = 0.05 \Leftrightarrow 5(1 - x^3) = 0.05 \Leftrightarrow x^3 = 0.9 \Leftrightarrow x = 0.9^{1/3} = 0.9655 \)