Erdős-Woods Numbers are defined as the length of an interval of consecutive integers whose every element is not coprime with its extremities. Woods was the first to find such numbers, Dowe proved there exists an infinity and Cegielski, Heroult and Richard proved their set is recursive. Our aim is to study the arithmetical proprieties of those numbers.
The first three members of the \( n \) infinite family:

\[(2n - 2, 2n - 1, (2n, 2n - 2))\] (1215, 1216).

For primes less than 23 there are four ambiguous cases for \( k = 2 \):

Perhaps \( k = 3 \):

\[x + 1, x + 2, \ldots, x + k\]

Conjecture: There is a positive integer \( k \) such that every \( x \) is uniquely determined by the list of prime divisors of

**History and Conjecture**
\[ \forall a, \exists p \left( a + c + p < a \right) \]

In other words: there exists a natural number \( c \) such that \( a > c + a + p \) and \( c \) is coprime with \( a \) and with \( a + p \).

Conjecture (Woods 1981): For any ordered pair \( (a, p) \) of natural numbers, with \( p < 3 \), there exists a natural number \( c \) such that \( a > c + a + p \) and \( c \) is coprime with \( a \) and with \( a + p \).
Examples to the Woods' conjecture:

**Theorem (Dowe 1989)**: There exists an infinity of such count-

<table>
<thead>
<tr>
<th>2,3</th>
<th>2,7</th>
<th>3,17</th>
<th>3,31</th>
<th>5,19</th>
<th>5,31</th>
<th>7,19</th>
<th>7,31</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,9</td>
<td>2,137</td>
<td>2,191</td>
<td>2,193</td>
<td>2,197</td>
<td>2,199</td>
<td>2,200</td>
<td>2,202</td>
</tr>
</tbody>
</table>

**Counterexample (Woods)**: 2184
Remark: $p$ can be a prime.

Idea: Primality of consecutive primes.

**Proposition 2**: $a$ and $p + a$ are composite numbers.

Idea: Primality of consecutive primes.

**Proposition 1**: If $p$ is an Erdős-Woods number, there exists two

**Elementary Propositions**
Ideas: Theorem

un nombre de Erdős-Woods on a p > a.

Proposition 4: Pour tout couple d'entiers a et b, tel que p soit

Idea: For every prime p', \( p' \neq 1 \), belongs to this set.

Woods number is infinite.

Proposition 3: The complementary set in \( \mathbb{N} \) of the set of Erdős-

Elementary Propositions
arithmetic progressions.

Idea: Analogous to the Cantor Theorem on consecutive primes in

\[ B \subset \{ \ldots, p^d \} \text{ if } B \subset \{ \ldots, p^d \} \text{ we have } p > (\ldots, p^d + \ldots) \text{ for } i \}

\[ p + a | (\ldots, p^d) \text{ and } i - p | (\ldots, p^d) \text{ then } B \subset \{ \ldots, p^d \} \text{ if } a | (\ldots, p^d) \text{ and } p > \ldots \text{ for any integer } i \}

\[ (\ldots, p^d) \text{ and } p \text{ from } \{ \ldots, p^d \} \text{ and an application from } \{ \ldots, p^d \} \text{ of primes (strictly) less than } p \text{ in two sets.}

An integer } d \text{ is an Erdős-Woods number if and only if there exist a

\text{Theorem (Cegieliski-Heroult-Richard 2000)}
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97,
101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197,
199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 290, 293, 299,
307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421,
431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541, 547, 557,
563, 569, 571, 577, 587, 593, 599, 601, 607, 613, 617, 619, 631, 641, 643, 647, 653, 659, 661, 673,
677, 683, 691, 697, 701, 709, 719, 727, 733, 739, 743, 751, 757, 761, 769, 773, 787, 797, 809, 811,

define Erdos-Woods numbers

First Erdos-Woods numbers

<table>
<thead>
<tr>
<th>Erdos-Woods numbers</th>
</tr>
</thead>
</table>
Perhaps \( \frac{\rho(n)}{n} \rightarrow \frac{1}{4} \)

More precisely: \( \rho(n) = O(n) \)?

Problem: Is the density \( \rho(n) \) of Erdös-Woods numbers linear?

\( \frac{\text{EW}(n)}{n} \sim f(n) \).

Problem: Find a (simple) function \( f \) such that \( f \) such that

Let \( \text{EW}(n) \) be the \( n \)-th Erdös-Woods number. \( \text{EW}(2) = 22, \text{EW}(3) = 34, \ldots \), \( \text{EW}(1) = 16 \).
There exists an infinity of twin Erdős-Woods numbers.

Conjecture (CHR 2000):

Every Erdős-Woods number is even.

Conjecture (Dowe 1989): Every associated to Erdős-Woods numbers is even.
an Erdős-Woods number.

Conjecture (Ysemirnov 2000): Every square greater than 16 is a number. Every square greater than 16 is an Erdős-Woods number.

Conjecture (CHR 2000): For any integer \( k \), there exists an integer \( p \) such that \( d, d+2, d+4, \ldots, d+2k \) are Erdős-Woods numbers.
Corollary: 903, 2545, 4533, ... are Erdős-Woods numbers.

then $p$ is an Erdős-Woods number.

$$q^{p} = q^{p}, \quad q \geq p \in *$$

$$p < q^{p}, \quad q \geq p \in A \ast$$

$q^{p} - p$ is a prime factor of $q^{p}$, $q \geq p \in A \ast$

$$\tau = 0p \ast$$

such that:

Theorem: If for an odd integer number $p$, there exists a sequence:

Results
Ideas
Remark: Vsemirnov's conjecture is false for 262 and 342.

Then \( q \) is an Erdős-Woods number.

\[
\zeta q = \zeta p, \quad q \not| p \in \ast
\]

\[
\zeta q < \zeta p, \quad q \not| p \in \ast
\]

\[
\zeta q - \zeta p = \zeta q - \zeta p - 1
\]

\[
1 + p = \zeta p \ast
\]

\[
1 = \psi p \ast
\]

\[\text{such that } \exists \text{ a sequence: } S_1, S_2, \ldots, S_n \text{ such that } p = S_1 S_2 \ldots S_n.\]

Theorem: If \( q \) is an even integer number with \( p + 1 \) prime, then there exist a sequence: \( S_1, S_2, \ldots, S_n \), such that \( p = S_1 S_2 \ldots S_n.\)
Corollaries and Certificates

Corollary: 16, 36, 84, 216, 21764, 484, 324, 256, 144, 100, 900, 1296, 1600, 256, 179, 139, 75, 39, 49, 30, 29, 6.

Certificates: 16, 36, 216, 2704, ... are Erdős-Woods numbers.

1764, 2116, 2704, ... are Erdős-Woods numbers.
16 510, [131, 68], 83, 29, [2621, 1506], [53, 3], [241, 138], 821, 667, [269, 230], 487, 192, 739, 2704, [[53, 3], [241, 138], 821, 667, [269, 230], 487, 192, 739, 2704, [[53, 3], [241, 138], 821, 667, [269, 230], 487, 192, 739, 2704, [[53, 3], [241, 138], 821, 667, [269, 230], 487, 192, 739, 2704, [[53, 3], [241, 138], 821, 667, [269, 230], 487, 192, 739, 2704, [[53, 3], [241, 138], 821, 667, [269, 230], 487, 192, 739, 2704]]

2116, [47, 4], [2069, 1068], [43, 11], [721, 403], [1764, 43, 41], [1721, 403], [1600, 41, 17], 1599, 1055, 1296, [37, 17], 1259, 989, 900, [[31, 3], 79, 52], 821, 364, 784, [[29, 14], 151, 40], [411, 16], 159, 161, [191, 41], [593, 41], [593, 15], [593, 41], 484, [23, 4], 461, 206, Certificates
17

\[ n = 9: 4032, 4034, 4036, 4038, 4040, 4042, 4044, 4046, 4048 \]

\[ n = 8: 4312, 4314, 4316, 4318, 4320, 4322, 4324, 4326 \]

\[ n = 7: 2166, 2168, 2170, 2172, 2174, 2176, 2178 \]

\[ n = 6: 1834, 1836, 1838, 1840, 1842, 1844 \]

\[ n = 5: 532, 534, 536, 538, 540 \]

\[ n = 4: 216, 218, 220, 222 \]

\[ n = 3: 92, 94, 96 \]

\[ n = 2: 34, 36 \]