STAT 145 (Notes)

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PRACTICE EXAM 1

SOLUTIONS TO SELECTED PROBLEMS.
Problem 17

The average salary of all female workers at a large plant is $35,000. The average salary of all male workers at the plant is $41,000. If there are more male workers than female workers at the plant, then the average salary at the plant must be ...

Solution
Let us assume that the number of female workers is 1 and the number of male workers is 2. Moreover, suppose that our observations are: 35,000 (female worker) and 41,000 and 41,000 (male workers).

\[ \bar{x} = \frac{35,000 + 41,000 + 41,000}{3} = 39,000 \]

We could repeat the same analysis for different numbers of female and male workers and we would reach the conclusion that the average salary at the plant must be larger than $38,000.
Problems 18 and 19
Use the following density curve to answer questions 18 and 19.
Problem 18

For the above density curve, the third quartile is:

\[ Q_3 = 1.5 \]

\[ \text{proportion} = 1.5 \times 0.5 = 0.75 = 75\% . \]

\[ Q_3 = 1.5 \]
Problem 19

For the above density curve, what percent of observations lie between 0.25 and 0.50:

\[
\text{percent} = (0.5 - 0.25) \times 0.5 = 0.125 = 12.5\%
\]
Problem 20

Scores on a University exam are normally distributed with a mean of 68 and a standard deviation of 9. Using the 68-95-99.7 rule, what percentage of students score above 77?

Solution

Let $X$ represent the score of a randomly selected student. We want to find $P(X > 77)$. Note that $77 = \mu + \sigma = 68 + 9$. 
Solution (cont.)

\[ P(X > 77) = 16\% \]
Problem 21

The time to complete a standardized exam is approximately normal with a mean of 70 minutes and a standard deviation of 10 minutes. Using the 68-95-99.7 rule, if students are given 90 minutes to complete the exam, what percentage of students will not finish?

Solution
Let $X$ represent the time a randomly selected student takes to complete a standardized exam. We want to find $P(X > 90)$. Note that $90 = \mu + 2\sigma = 70 + 2(10)$. 

Solution (cont.)

\[ P(X > 90) = 2.5 \% \]
Problem 22

Using the standard normal distribution tables, what is the area under the standard normal curve corresponding to $Z < 1.15$?
Solution

To find the area to the left of 1.15, locate 1.1 in the left-hand column of Table A, then locate the remaining digit 5 as 0.05 in the top row. The entry opposite 1.1 and under 0.05 is 0.8749. This is the area we seek.
Problem 23

The scores on a university examination are normally distributed with a mean of 62 and a standard deviation of 11. If the bottom 5% of students will fail the course, what is the lowest mark that a student can have and still be awarded a passing grade?
Solution

1. State the problem. We want to find the score $x^*$ with area 0.05 to its left under the Normal curve with mean $\mu = 62$ and standard deviation $\sigma = 11$.

2. Use the table. Look in the body of Table A for the entry closest to 0.05 to its left. It is 0.0505. This is the entry corresponding to $z^* = -1.64$. So $z^* = -1.64$ is the standardized value with area 0.05 to its left.

3. Unstandardize to transform the solution from the $z^*$ back to the original $x^*$ scale. We know that the standardized value of the unknown $x^*$ is $z^* = -1.64$.

So $x^*$ itself satisfies

$$\frac{x^* - 62}{11} = -1.64$$

Solving the equation for $x^*$ gives

$$x^* = 62 + (-1.64)(11) = 43.96$$
Problem 25

Chocolate bars produced by a certain machine are labeled as 8.0 oz. The distribution of the actual weights of these chocolate bars is normal with a mean of 8.1 oz. and a standard deviation of 0.1 oz. The proportion of chocolate bars with weights between 8.2 and 8.3 oz. is ...
Solution

Let $X$ be the weight of a randomly selected chocolate bar. We have that $X$ has a Normal distribution with mean $\mu = 8.1$ and $\sigma = 0.1$. We would like to find:

\[
P(8.2 < X < 8.3) = P(X < 8.3) - P(X < 8.2)
\]

\[
P(8.2 < X < 8.3) = P\left(\frac{X-\mu}{\sigma} < \frac{8.3-8.1}{0.1}\right) - P\left(\frac{X-\mu}{\sigma} < \frac{8.2-8.1}{0.1}\right)
\]

\[
P(8.1 < X < 8.2) = P(Z < 2) - P(Z < 1) = 0.9772 - 0.8413
\]

\[
P(8.2 < X < 8.3) = 0.1359
\]