Laplace Transform Instructions

Shift Formulas
- \( \mathcal{L}[e^{at}f(t)] = F(s-a) \). To take the Laplace transform of \( e^{at}f(t) \):
  1. Find \( \mathcal{L}[f(t)] = F(s) \).
  2. Replace \( s \) with \( s-a \) in \( F(s) \).

- \( \mathcal{L}^{-1}[F(s)] = e^{at}\mathcal{L}^{-1}[F(s+a)] \). To take the inverse Laplace transform of \( F(s) \):
  1. Replace \( s \) with \( s+a \) in \( F(s) \).
  2. Take the inverse Laplace transform of \( F(s+a) \)
  3. Multiply by \( e^{at} \).

- \( \mathcal{L}[u_c(t)f(t)] = e^{-cs}\mathcal{L}[f(t+c)] \). To take the Laplace transform of \( u_c(t)f(t) \):
  1. Replace \( t \) with \( t+c \) in \( f(t) \).
  2. Take the Laplace transform of \( f(t+c) \).
  3. Multiply by \( e^{-cs} \).

- \( \mathcal{L}^{-1}[e^{-cs}F(s)] = u_c(t)f(t-c) \). To take the inverse Laplace transform of \( e^{-cs}F(s) \):
  1. Find \( \mathcal{L}^{-1}[F(s)] = f(t) \)
  2. Replace \( t \) with \( t-c \) in \( f(t) \).
  3. Multiply by \( u_c(t) \).

Differentiation/Integration Formulas
- \( \mathcal{L}\left[t^n f(t)\right] = (-1)^n \frac{d^n}{ds^n} F(s) \). To take the Laplace transform of \( t^n f(t) \):
  1. Find \( \mathcal{L}\left[f(t)\right] = F(s) \).
  2. Take \( n \) derivatives of \( F(s) \) with respect to \( s \).
  3. Multiply by \( (-1)^n \).

- \( \mathcal{L}^{-1}\left[\frac{1}{s} F(s)\right] = \int_0^t f(u)du \). To take the inverse Laplace transform of \( \frac{1}{s} F(s) \):
  1. Find \( \mathcal{L}^{-1}[F(s)] = f(t) \).
  2. Integrate \( f(t) \) from 0 to \( t \).

Convolution Formula
- \( \mathcal{L}^{-1}\left[F(s)G(s)\right] = f(t) \ast g(t) \). To take the inverse Laplace transform of \( F(s)G(s) \):
  1. Find \( \mathcal{L}^{-1}[F(s)] = f(t) \) and \( \mathcal{L}^{-1}[G(s)] = g(t) \).
  2. Convolute \( f(t) \) and \( g(t) \).