1. (25pts.) Find the inverse Laplace transforms
   
   (a) (5pts)
   \[ F(s) = \frac{1}{s(s - 2)(s - 3)} \]
   
   (b) (5pts)
   \[ F(s) = \frac{1}{s^2(s^2 - 1)} \]
   
   (c) (5pts)
   \[ F(s) = \frac{2s}{s^2 + 2s + 2} \]
   
   (d) (5pts)
   \[ F(s) = \frac{5s - 4}{s^3 - s^2 - 2s} \]
   
   (e) (5pts)
   \[ F(s) = \frac{s^2 + 1}{(s^2 + 2s + 5)^2} \]
   
2. (15pts.) Solve the IVP
   \[ \frac{dx}{dt} = 2x - x^2 \ , \ x(0) = 1 \]
   and sketch the direction field and the solution.

3. (12pts.) Give the general solution for the ODE
   \[ \frac{d^2y}{dt^2} + 8\frac{dy}{dt} + k^2y = 0 \]
   when the spring constant \( k \) has the values:
   
   (a) \( k = 5 \).
   
   (b) \( k = 4 \).
   
   (c) \( k = \sqrt{7} \).

   In each case, state if the system is overdamped, critically damped, or underdamped

4. (14pts.) Solve the IVP
   \[ \frac{dy}{dt} + \frac{2t}{t^2 + 1}y = \frac{1}{t} \ , \ y(1) = 0 \]
   and identify the largest interval in \( t_1 < t < t_2 \) for which the solution is defined.
5. (6pts.) Compute the Wronskian of the functions \( y_1(x) = e^x \) and \( y_2(x) = e^{-x} \). Are these linearly independent?

6. (14pts.) Solve the IVP using Laplace transforms:

\[
\frac{d^2y}{dt^2} + y = \cos t, \quad y(0) = y'(0) = 0.
\]

7. (14pts.) Solve the IVP using Laplace transforms:

\[
\frac{d^2y}{dt^2} + 3 \frac{dy}{dt} + 2y = e^{-t}, \quad y(0) = 0, y'(0) = 1.
\]