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Twelfth New Mexico Analysis Seminar

Department of Mathematics and Statistics
University of New Mexico
Albuquerque
April 23-25, 2009

ABSTRACTS

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Keynote Speaker: Loukas Grafakos, University of Missouri-Columbia

Lectures on *Linear and Multilinear Singular Integrals I, II, III*

Outline:

- Calderón-Zygmund Operators (linear and m-linear).
- The Calderón-Zygmund theorem.
- The method of rotations.
- Directional Hilbert transforms and the Bilinear Hilbert Transform.
- Calderón commutators.
- Recent results on the commutators.
- m-linear multiplier Theorems (Hörmander, Coifman-Meyer).
- The m-linear $T1$ theorem.
- Discretization and almost diagonal conditions.

Time permitting: Maximal singular integral operator and discussion on weights. "More singular" singular integrals in the linear case and also in the bilinear case (those associated with a function Omega). Return to the bilinear Hilbert transform-a small discussion. The disc multiplier (linear and bilinear). Counterexamples for the disc multiplier in higher dimensions

Invited Speakers

- Arpad Benyi, Western Washington University
  Bilinear Pseudodifferential Operators: the Coifman-Meyer class and beyond
  Abstract: Bilinear (or multilinear) operators can be viewed as generalizations of the product of functions and their derivatives. In this talk, we introduce several natural classes of bilinear pseudodifferential operators, investigate their continuity properties on various function spaces such as Lebesgue, Sobolev and modulation spaces, and present a generic symbolic calculus. We will conclude the presentation with a $T(1,1)$-theorem applicable to bilinear operators beyond the Coifman-Meyer class that obey a certain modulation invariance.
- Diego Maldonado, Kansas State University
  Paraproducts and the Bilinear Calderón-Zygmund Theory
  Abstract: We will introduce several, essentially equivalent, definitions of paraproducts, mention some of their applications to Analysis and PDEs, illustrate their connections to the bilinear Calderón-Zygmund theory, and derive some of their mapping properties.

Contributed Talks

- Dhanapati Adhikari (Oklahoma State University, Stillwater, OK)
  Title: *Global Regularity Problem for the 2D Boussinesq Equations.*
  Abstract: The Boussinesq system of equations models the thermal convection and geostrophic flows. Global regularity problem of the inviscid Boussinesq equation is an outstanding open problem. We will briefly mention about the recent result on global regularity of 2D Boussinesq system. We will discuss the global regularity of classical solutions of the 2D Boussinesq equation with the vertical viscous dissipation and the vertical diffusion.

- Marco Annoni (University of Missouri-Columbia, MO)
  Title: *Almost everywhere convergence of modified Bochner-Riesz means at the critical index*
  Abstract: Let $m_{\lambda,\gamma}$ be the Bochner-Riesz means modified by Andreas Seeger:

  $$m_{\lambda,\gamma}(\xi) = \frac{(1-|\xi|^2)_+^\lambda}{(1-\log(1-|\xi|^2))^\gamma}$$

  where $\xi \in \mathbf{R}^n$. For any real number $R > 0$, let $(m_{\lambda,\gamma})_R$ denote its dilation: $(m_{\lambda,\gamma})_R(\xi) = m_{\lambda,\gamma}(\varphi \xi R)$. Let us define the operator $B_R^{\lambda,\gamma}$ via the mean $(m_{\lambda,\gamma})_R$ by: $B_R^{\lambda,\gamma}(f)(x) = \left(\widehat{f} \cdot (m_{\lambda,\gamma})_R\right)^\vee(x)$ where $x \in \mathbf{R}^n$. I'll show that, for γ big enough, we have:

  $$\lim_{R\to\infty} B_R^{\lambda,\gamma}(f)(x) = f(x)$$

  for almost every $x \in \mathbf{R}^n$, if $f \in L^{p_\lambda}(\mathbf{R}^n)$, for the critical index $p_\lambda = \frac{2n}{n-1-2\lambda}$.

- María Pía Becca-Varela (New Mexico State University, Las Cruces, NM)
  Title: *Long correlations and Levy Models applied to the study of Memory effects in high frequency (tick) data.*
  Abstract: We study the stochastic differential equation modeling the evolution of high frequency data. We use a sample of 25 stocks for this purpose. We verify that the behavior of the return is compatible with that of continuous time Levy processes. We also study the presence of memory effects and long-range correlations in the values of the return.

- Oleksandra Beznosova (University of Missouri-Columbia, MO)
  Title: *Linear, with respect to the A_2-constant of the weight w, bound on the norm of the perfect dyadic operator on weighted Lebesgue spaces $L^2(w)$.*
  Abstract: We prove a sharp version of the T(1) theorem for the the perfect dyadic singular integral operators on the real line.

- Matthew Blair (University of New Mexico, Albuquerque, NM)
  Title: *Strichartz Estimates for the Schrödinger Equation in Exterior Domains.*
  Abstract: Strichartz estimates are a family of space-time integrability estimates for the Schrödinger equation that rely on the dispersive effect of the solution map. Generally speaking, these estimates are well-understood when the equation is posed over Euclidean space. However, the situation is much more complicated when one starts to consider the obstacle problem, as the local and global geometry of the boundary can influence how waves develop. We will survey recent parametrix constructions that yield these inequalities, with emphasis on a recent joint work with H. Smith and C. Sogge. A key feature of such constructions is the use of related local smoothing estimates which handle the error terms.

- Xi (Ron) Cheng (University of New Mexico, Albuquerque, NM)
  Title: *Scattering from a lossless sphere*
  Abstract: We study fundamental issues in electromagnetic scattering theory, with an emphasis on pole behaviors of a lossless sphere arising from the singularity expansion method (SEM). We use Mie Theory to solve the acoustic and electromagnetic scattering problems for spheres with lossless boundary conditions and an incident plane wave. We show that for certain lossless sheet impedance boundary conditions there exist second order poles for both cases. Our general procedure to directly construct lossless sheet impedance boundary conditions which will produce high order poles is discussed as well as the difficulties to which it leads. In the electromagnetic scattering case, Foster Rs Theorem is imposed on the impedance condition to ensure that a lossless scattering problem is obtained. We also study the validity of the forward-scattering theorem associated with SEM.

- Pavlo Cherepanov (University of New Mexico, Albuquerque, NM)
  Title: *Comparison of Shock Forming Properties of Continuum and Kinetic Models*
  Abstract: Continuum Mechanics and Kinetic Theory are two mathematical theories with fundamentally different approaches to the same physical phenomenon. Continuum Mechanics together with Thermodynamics treat a substance (a gas or a fluid) as a continuous medium and describes the evolution of its macro characteristics via application of the Conservation Laws to small packets of the substance. Kinetic Theory attempts to describe the evolution of the macro parameters by treating a substance as a family of colliding objects. The number of objects must be large enough so a statistical approach can be taken. In this work we introduce a numerical scheme to solve 1-D Bhatnagar-Gross-Krook (BGK) model equations and examine the formation of a stationary viscous shock. Obtained results are compared to a stationary numerical solution of 1-D Navier-Stokes equation with a similar set of shock forming conditions.

- Daewon Chung (University of New Mexico, Albuquerque, NM)
  Title: *On a linear bound for the commutator of the Hilbert transform on weighted Lebesgue space.*
  Abstract: The operator norm of the Hilbert transform can be replaced by the norm of the dyadic shift operator. We prove the linear bound on the norm of the commutator of the Hilbert transform, in the weighted Lebesgue space $L^2(w)$, with respect to the A_2-characteristic of the wieght w, by estimating the norm of the commutator of the dyadic shift operator with BMO functions. With this result, we can extrapolate to $L^p(w)$.

- Geraldo De Souza (Auburn University, Auburn, AL)
  Title: *A new look on the Lorentz Spaces $L(p,1)$ for $1 < p < \infty$, and Applications*
  Abstract: In 1950 G. G. Lorentz introduced in his paper entitled "Some New Functional Spaces" at Annals of Mathematics, the functional spaces denoted by $\Lambda(\alpha)$ for $0 < \alpha < 1$, and defined as the set of real measurable functions $f(x)$, $0 < x < 1$ for which

  $$\|f\|_{\Lambda(\alpha)} = \alpha \int_0^1 x^{\alpha-1} f^*(x)\, dx,$$

  where $f*$ is the decreasing rearrangement of f.

  In this talk we give two simple new characterizations of $\Lambda(1/p)$ for $1 < p < \infty$, based on generalizations of the special atoms spaces, introduced by the author on his earlier works. The spaces $\Lambda(1/p)$ is now days denoted by $L(p,1)$. We use these characterizations to give a rather simple proof of Weiss-Stein theorem on the extension of operators on $L(p,1)$ and other applications.

- Cesar Garcia (New Mexico State University, Las Cruces/ITAM, Mexico)
  Title: *Variations on Ekeland's Variational Principle.*
  Abstract: In this talk we present some extensions of Ekeland's variational principle to locally convex topological vector spaces. We discuss some reformulations of this principle and plausible extensions of them.

- Liguang Liu (University of Missouri-Columbia, MO/Beijing Normal University, China)
  Title: *BLO-type spaces associated with the Ornstein-Uhlenbeck operator.*
  Abstract: In the setting of Gauss measure metric space, we introduce some BLO-type space (i.e., the space of functions with bounded lower oscillation), and investigate some relations between the known BMO-type space of Mauceri and Meda and such BLO spaces. Also, boundedness of maximal singular integral operators, including imaginary powers of the Ornstein-Uhlenbeck operator and Riesz transforms of any order associated with the Ornstein-Uhlenbeck operator, are analyzed.

- Farid Madani (Pierre and Marie Curie University (Paris 6), France)
  Title: *Yamabe type equations*
  Abstract: The Yamabe equation is a nonlinear elliptic PDE. It appears, when we try to solve a geometric problem on compact riemannian manifolds. We generalize the Yamabe type equations for a larger class of functions and we will give a sufficient condition to solve them.

- Kabe Moen (Kansas University, Lawrence, KS)
  Title: *Weighted Inequalities for multilinear fractional integral operators.*
  Abstract: A weighted theory for multilinear fractional integral operators and maximal functions is presented. Sufficient conditions for the two weight inequalities of these operators are found, including "power and logarithmic bumps" and an A_∞ condition. For one weight inequalities a necessary and suffcient condition is then obtained as a consequence of the two weight inequalities. As an application, Poincaré and Sobolev inequalities adapted to the multilinear setting are presented.

- Emmanuel Ncheuguim (New Mexico State University, Las Cruces, NM)
  Title: *Non linear Black-Scholes option pricing models.*
  Abstract: The study of Black-Scholes type model for option pricing with transaction costs leads to nonlinear parabolic equations. Under suitable conditions, we prove the existence and the uniqueness of such problems and investigate some regularity and stability results.

- Olena Ostapyuk (Kansas State University, Manhattan, KS)
  Title: *Convergence of backward-iteration sequences with bounded hyperbolic step in higher dimension.*
  Abstract: I consider a holomorphic self-map f of the unit ball in $\mathbb{C}^N$, of hyperbolic type (with a dilatation coefficient $c < 1$ at the Denjoy-Wolff point of f). I have shown that any backward-iteration sequence with bounded hyperbolic step must converge to some point on the boundary other than the Denjoy-Wolff point and stay in a Koranyi region. The proof is based on the multi-dimensional version of Julia's lemma. When $N = 1$ these limit points are known to be boundary repelling fixed points for f. I will generalize this notion for $N > 1$.

- Mohammed Qazi (Tuskegee University, Tuskegee, AL)
  Title: *A Question Concerning a Polynomial Inequality and an Answer.*
  Abstract: Let $\mathcal{P}_n$ be the class of all polynomials of degree at most n, and let $\mathcal{M}_p(g\,;\,\rho)$ denote the L^p mean of g on the circle of radius ρ centered at the origin. We specify a number $\rho^* \in (0\,,\,1)$, depending on n and k, such that for any $f \in \mathcal{P}_n$, the ratio $\mathcal{M}_p(f^{(k)}\,;\,\rho)/\mathcal{M}_p(f\,;\,1)$ is maximized by $f(z) := z^n$ for all $\rho \in [\rho^*\,,\,\infty)$ and $p \geq 1$. Here, $f^{(k)}$ denotes the k-th derivative of f. The interest of the result lies in the fact that ρ^* is strictly less than 1.

- Yan Qiu (University of New Mexico, Albquerque, NM)
  Title: *A non-linear Black-Scholes equation.*
  Abstract: We study a modification of the Black-Scholes equation allowing for uncertain volatility. The model leads to a partial differential equation with nonlinear dependence upon the highest derivative. Under certain assumptions, we show existence and uniqueness of a solution to the Cauchy problem.

- Marc Salas (New Mexico State University, Las Cruces, NM)
  Title: *Hydrodynamic Models for Charged Transport in Semiconductors.*
  Abstract: We study a quasilinear parabolic and elliptic system of partial differential equations. This system models the time evolution of a one-dimensional particle distribution for charged particles within a semiconductor device. We decouple the system and study the existence and uniqueness of solutions to the Cauchy-Dirichlet problem for the resulting quasilinear parabolic integro differential equation. Using a priori estimates, we obtain global existence and uniqueness results for a more general problem.

- Lijing Sun (University of Wisconsin-Milwaukee, WI)
  Title: *Certain Oscillating Operator Related to Wave Equations in Block Spaces.*
  Abstract: We give a sufficient condition for certain oscillating operator to extend to a bounded operator in Block space.

- Oliver Thomys (Leipzig University, Germany)
  Title: *On the Uniqueness of the Solution of the Capillary Problem in Porous Materials*
  Abstract: The behaviour of capillary effects in porous materials is often considered in physical chemistry. Here the qualitative ascent of fluids on the walls of the pores is examined. The principal point is the extension of a comparison principle of Concus and Finn on the current circumstances. So for straight cylindrical cavities with arbitrary cross-section a comparison principle is obtained. For some special cases (circular cross-section, horizontal wall, parallel plates) explicit results are stated.

- Ray Treinen (Kansas State University, Manhattan, KS)
  Title: *Floating Drops and Functions of Bounded Variation.*
  Abstract: A drop of one fluid is collected on the interface between two other fluids. Properties of the equilibrium configuration are sought. Elcrat, Neel, and Siegel gave first variation argument that led to studying the configuration with ODE's. This approach left some open problems. We attempt to address these issues using the theory of functions of bounded variation.

- Wilfredo Urbina (De Paul University, Chicago, IL)
  Title: *Boundedness of Fractional Integrals and Fractional Derivatives on Gaussian Lipschitz spaces.*
  Abstract: We consider Lipschitz spaces with respect to the Gaussian measure $Lip_\alpha(\gamma_d)$, and study the boundedness of fractional integral and fractional derivative operators on them. The methods are general enough to provide alternative proofs of the ones given in the classical case and moreover can be extended to the case of Lagerre expansions and Jacobi expasions too.

- Dmiter Vassilev (University of New Mexico, Albuquerque, NM)
  Title: *Some interesting classes of quaternion functions.*
  Abstract: We consider certain classes of so called anti-regular functions of quaternionic variables and corresponding classes of functions on manifolds with quaternionic structures. We consider their applications to questions concerning geometric partial differential equations.
- Ana Luz Vivas-Mejia (New Mexico State University, Las Cruces, NM)
  Title: *Existence of Solution to an Integro-Differential Parabolic Problem arising on Financial Mathematics.*
  Abstract: In the standard Black-Scholes model, a basic assumption is that the volatility is constant. Several models that have been proposed in recent years, however, allowed the volatility to be an stochastic variable, whith a standard Brownian motion included in the model. The Black-Scholes models with jumps arise in the fact that the Brownian Random Walk doesn't fit the financial data presenting large fluctuations; The necessity of taking into account large market movements, and a great amount of information arriving sudenly (i.e. a jump) has led to the study of partial integro-differential equations (PIDE) in which the integral term is modeling the jump. We study the existence of solutions in a general
- Fayou Zhao (University of Missouri-Columbia, MO/Beijing Normal University, China)
  Title: *CBMO Estimates for Multilinear Hardy Operators*
  Abstract: In 1976, Faris defined the n-dimensional Hardy operators on $\mathbb{R}^n$. In 1995, Christ and Grafakos gave a generalized version of the classical Hard y's inequality in $\mathbb{R}^n$. In 2007, Fu et al. considered the boundedness of commutators generated by central BMO functions and N-dimension fractional Hardy operators. The purpose of this paper is to study the multilinear Hardy operators in higher dimensional case and establish the CBMO estimates for multilinear Hardy operators on some function spaces, such as the Lebesgue spaces, the Herz spaces and the Morrey-Herz spaces.

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Organizers: Tiziana Giorgi (NMSU), Joseph Lakey (NMSU),
Cristina Pereyra (UNM), Robert Smits (NMSU).