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SEVENTH NEW MEXICO ANALYSIS SEMINAR

Department of Mathematics and Statistics
University of New Mexico, Albuquerque
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ABSTRACTS

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MINICOURSES ABSTRACTS

- Patricia Bauman (Purdue University)
  Title: *Analysis of solutions to Ginzburg-Landau Models.*
  Abstract: In this minicourse we describe mathematical techniques for analyzing the behavior of solutions to the Ginzburg-Landau system describing superconducting materials in an applied magnetic field. We also discuss models including pinning, and connections with the Landau-de Gennes models for phase transitions in smectic liquid crystals.

- Luca Capogna (University of Arkansas)
  Title: *Mean curvature flow in the Heisenberg group.*
  Abstract: The Heisenberg group is among the simplest examples of Sub-Riemannian manifold. Such structure involves a metric which is defined only on a subspace of the tangent bundle. Special properties of the sub-bundle (like non-integrability) guarantee the existence of a well-defined control metric.

  Examples of sub-Riemannian manifolds come from different fields, e.g. the tangent space in the tangent bundle of a CR manifold, the admissible directions in optimal control problems, etc., etc.

  The Heisenberg metric allows to define a notion of perimeter, which was first introduced by Pierre Pansu in 1980, in his proof of the Heisenberg isoperimetric inequality.

  The first variation of perimeter along the "admissible" directions describes a geometric flow, in which the perimeter decreases and the velocity is admissible. By analogy with the Riemannian setting we call such flow: The Mean Curvature Flow.

  In the course of two lectures we will present basic background definitions, describe the first variation and derive the non-linear, degenerate parabolic PDE system which governs the flow.

Subsequently we will investigate the existence of self-similar solutions, the short-time existence and study the evolution of the associated Legendrian foliation. If there is time we will talk about weak (viscosity) solutions and discuss asymptotic behavior.

The mean curvature flow has obvious relevance in the study of minimal surfaces and in discovering what is the isoperimetric profile of the Heisenberg group.

INVITED LECTURES ABSTRACTS

- Peter Sternberg (Indiana University)
  Title: *Elliptic Variational Problems on Constricted Networks with Applications to Ginzburg-Landau Theory*
  Abstract: I will analyze variational problems set on a network of thin constricted tubes. In the asymptotic regime where the tubes collapse to a graph, one can identify a one dimensional variational problem with interesting natural boundary conditions at the nodes which in particular encourage jumps in the (limit of) minimizers across the various branches of the graph. An application to tunneling across weak links in a superconductor will be discussed.

- Donatella Danielli (Purdue University)
  Title: *Hypersurfaces of minimal type in sub-Riemannian geometry.*
  Abstract: In this talk I will develop a sub-Riemannian calculus on hypersurfaces in Carnot groups and discuss some applications. One of them is a partial solution of the isoperimetric problem for the Heisenberg group.

- Lia Bronsard (McMaster University)
  Title: *Fractional degree vortices for a spinor Ginzburg-Landau model.*
  Abstract: Mathematicians often speak of "the" Ginzburg-Landau model, but variational models of Ginzburg-Landau type abound in condensed matter physics, to give macroscopic or mesoscopic descriptions of systems exhibiting various types of ordering, such as superconductivity, superfluidity, ferromagnetism, or smectic ordering in liquid crystals, for example. In this talk we present a spin-coupled (or spinor) Ginzburg-Landau model with complex vector-valued order parameter. Similar models have been introduced in the physics literature in order to account for ferromagnetic or antiferromagnetic effects in high-temperature

superconductors and in optically confined Bose–Einstein condensates. I will show how such models give rise to new types of vortices, with fractional degree and non-trivial core structure, and illustrate the various possibilites with some specific examples of Dirichlet problems in the unit disk. This talk represents works done in collaboration with S. Alama and P. Mironescu.

- Scott Pauls (Dartmouth College)
  Title: *Representations of Minimal Surfaces in the Heisenberg group with applications to the Bernstein Problem.*
  Abstract: We will discuss a geometric characterization of minimal surfaces in the three dimensional sub-Riemannian Heisenberg group as a special class of ruled surfaces. This characterization allows the development of adapted coordinates which yield a nice representation theorem for minimal surfaces in this setting. As an application, we will discuss solutions to analogues of the Bernstein problem. In particular, we will discuss a counterexample to the following conjecture:

  The only C^2 non-characteristic entire minimal graphs in the three dimensional Heisenberg group are vertical planes.

  In a positive direction, we will show that a C^2 connected minimal surface which is the graph over some plane must satisfy a type of constant curvature condition.

  The majority of this work is joint work with N. Garofalo.