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NINTH NEW MEXICO ANALYSIS SEMINAR

Department of Mathematics and Statistics
University of New Mexico
Albuquerque
April 6-8, 2006

ABSTRACTS

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MAIN LECTURES

Main speaker: **Tatiana Toro** (University of Washington, Seattle, WA)

Title: **Geometric measure theory as a tool in free boundary regularity problems**

Abstract: Free boundary problems arise naturally in physics and engineering. The free boundary may appear as the interface between a fluid and the air, or water and ice. In the filtration problem, which studies how water filtrates from a dam made of a porous medium (say earth), the free boundary separates the wet part from the dry part. The central problem of characterizing the regularity of the free boundary has been studied by many authors, among others Alt, Caffarelli, Friedman, Jerison, Kenig and Weiss, just to mention a few. Ideas from Geometric Measure Theory (GMT) play an important role in the understanding of the structure of the free boundary.

The goal of these lectures is to illustrate how ideas from GMT and potential theory come together to help us understand the structure of the free boundary for the Poisson kernel.

Our aim is that this series of lectures be self-contained. To achieve this we will briefly cover the following topics:

- Basic facts about harmonic and subharmonic functions
- Non-negative harmonic functions on non-tangentially accessible (NTA) domains
- Sets of locally finite perimeter.

This is roughly the content of the first two lectures. The purpose of the third lecture is to bring together all the concepts introduced above and present the proof of a joint result with Kenig which generalizes a well know theorem of Alt and Caffarelli. If time permits we will discuss Weiss' monotonicity formulas and their applications in this setting.

References

1. Alt & Caffarelli, *Existence and Regularity for a minimum problem with free boundary*, J. Reine Angew. Math. 325 (1981), 105–144.
2. Evans & Gariepy,*Measure Theory and Fine Properties of Functions*, Studies in Advanced Mathematics, CRC Press, 1992.
3. Kenig, *Harmonic Analysis Techniques for Second Order Elliptic Boundary Value Problems*, CBMS, 83, 1994.
4. Kenig & Toro, *Free Boundary Regularity for harmonic measures and Poisson kernels*, Ann. of Math. 150 (1999), 369–454.
5. Kenig & Toro,*Poisson Kernel Characterization of Reifenberg Flat Chord Arc Domains*, Ann. Scient. Ec. Norm. Sup. 36, (2003), 323-401.
6. Kenig & Toro, *On the free boundary regularity theorem of Alt and Caffarelli*, Discrete and Continuous Dynamical Systems 10, 2004, 397-422.

SHORT TALKS

- **Ivan Avramidi** (New Mexico Institute of Mining and Technology, Socorro, NM)
  Title: **Spectral asymptotics of non-commutative Laplacian.**
  Abstract: We study second-order elliptic self-adjoint partial differential operators acting on sections of a vector bundle over a manifold with non-commutative leading symbols. Such symbols generate a "matrix" generalization of Riemannian geometry when instead of a Riemannian metric there is a matrix valued self-adjoint symmetric two-tensor that plays a role of a "non-commutative" metric. We compute the first two coefficients of the asymptotic expansion of the trace of the corresponding heat kernel.

- **Geoff Diestel** (University of Missouri at Columbia, MO)
  Title: **The Multilinear Ball Multiplier Problem.**
  Abstract: For positive integers $k, n > 1$, the characteristic function of a smooth ellipsoid in $\mathbb{R}^{nk}$ is not the symbol of a bounded k-linear operator mapping $L^{p_1}(\mathbb{R}^n) \times L^{p_k}(\mathbb{R}^n)$ into $L^p(\mathbb{R}^n)$ if $1 < p_1, \cdots, p_k, p < \infty$, $1/p_1 + \cdots + 1/p_k = 1/p$ and exactly one of $p_1, \cdots, p_k, p'$ less than two.

- **Michael Eydenberg** (New Mexico State University, Las Cruces, NM)
  Title: **The Weyl Correspondence as a Functional Calculus.**
  Abstract: The Weyl calculus was developed in the efforts Hermann Weyl and other mathematical physicists to better understand the correlation between physical observables in classical mechanics and their quantum-mechanical analogues. Formally, it assigns to a classical observable $f : \mathbb{R}^{2n} \to \mathbb{R}^n$ defined on the state space (p, q) the operator and position operators on $L^2(\mathbb{R}^n)$. The existence of this operator and its representationdepends on the classes of functions f being considered. In this expository talk, I will show that this operator makes sense as a differential operator on $\mathcal{S}(\mathbb{R}^\backslash)$ when f is a polynomial. In particular, I demonstrate using commutators that the correspondence coincides with the natural definition for $f(P, Q)$ obtained from the spectral calculus.

- **Raluca Felea** (Rochester Institute of Technology, Rochester, NY)
  Title: **An FIO calculus for the marine seismic imaging: folds and cross caps.**
  Abstract: We are interested in a linearized inverse scattering problem considered by W. Symes and C. Nolan: the marine geometry. Airguns send acoustic waves through the ocean to the subsurface. The reflected rays are received by hydrophones towed behind a vessel. One uses the pressure field at the surface to reconstruct an image of the subsurface. Our goal is to invert F, the linearized operator which maps singularities in the sound speed to the resulting pressure field at the surface. We consider theoperator F^*F. Symes and Nolan proved that under no caustic assumption, F^*F is a pseudodifferential operator. We make the assumption that fold caustics occur and show that F is associated to a folded crosscap canonical relation and that the kernel of F^*F belongs to a class of distributions associated to two cleanly intersecting lagrangians.

- **Maria Cristina Mariani** (New Mexico State University, Las Cruces, NM)
  Title: **Essentially different and decaying periodic solutions of the forced pendulum equation with friction.**
  Abstract: This work is devoted to the study of the general forced pendulum equation in the presence of friction,
  $$u'' + a(t)u' + b(t)\sin u = f(t)$$
  with $a, b \in C([0, T])$ and $f \in L^2(0, T)$. We apply a Lyapunov-Schmidt reduction in order to obtain T-periodic solutions as zeroes of a 2π-periodic continuous real function under appropiate conditions on a, b and f. The multiplicity of solutions will be also studied.

- **Tao Mei** (Texas A & M University, College Station, TX)
  Title: **Matrix valued BMO spaces and paraproducts.**
  Abstract: The BMO (Bounded Mean Oscillation) Space, introduced by John and Nirenberg, plays an important role in classical analysis. It was characterized as the (real) dual space of the Hardy Spaces H_1 by Fefferman in 1971 and has immediate relations to paraproducts, Carleson measures, etc. I will talk about the generalization of the classical $H^1 - BMO$ duality theory for the matrix valued functions. If time permits, I will go further to talk about the matrix valued paraproducts.

- **Hamed Obiedat** (New Mexico State University, Las Cruces, NM)
  Title: **A Topological Characterization of the Beurling-Bjrck Space S_w Using the Short-Time Fourier Transform.**
  Abstract: We will use a previously obtained topological characterization of the Beurling-Björck space, to prove a topological characterization of the Beurling- Björck space via the short-time Fourier transform. Our work builds on recent work by K. Gröchenig and G. Zimmermann.

- **Christina Selby** (Purdue University, West Lafayette, IN)
  Title: **An extension and trace theorem for functions of G-bounded variation in Carnot Groups of step 2.**
  Abstract: An extension is given for a function $u \in BV_H(\Omega)$ to a function $u_0 \in BV_H(G)$ when Ω is "H-admissible," and G is a step 2 Carnot group. Here, BV_H is the class of functions of H-bounded variation. It is shown that H-admissible domains include non-characteristic domains and domains in groups of Heisenberg type which have a partial symmetry about characteristic points. An example is given of a domain that is $C^{1,\alpha}$, $\alpha < 1$, that is not H-admissible. Further, when Ω is H-admissible a trace theorem is proved for $u \in BV_H(\Omega)$.

- **Wilfredo Urbina** (Universidad Central de Venezuela/University of New Mexico, Albuquerque, NM)
  Title: **Orthogonal polynomials with Hermitian matrix argument.**
  Abstract: We construct and study orthogonal bases of generalized polynomials with argument on the space of Hermitian matrices.

- **Bixian Wang** (New Mexico Institute of Mining and Technology, Socorro,NM)
  Title: **Asymptotic Behavior of the FitzHugh-Nagumo System**
  Abstract: For the FitzHugh-Nagumo system defined on $\mathbb{R}$, we prove the existence of a compact global attractor in a weighted Sobolev space which contains bounded solutions, in particular, traveling wave and spatially periodic solutions. We also study the behavior of the global attractors as a parameter ϵ goes to zero. Although the limiting system for $\epsilon = 0$ does not possess a bounded attracting set, we show that there exists a constant ϵ_0 such that global attractors for $0 < \epsilon \leq \epsilon_0$ are all contained in a compact subset of the phase space. Furthermore, we construct a compact local attractor for the limiting system and establish the upper semicontinuity of the global attractors of perturbed system and the local attractor of the limiting system.

- **Michael Wilson** (University of Vermont, Burlington, VM)
  Title: **The intrinsic square function.**
  Abstract: We show that the Lusin area function and essentially all of its real-variable generalizations are pointwise dominated by an "intrinsic" square function, and that this latter function is, for all practical purposes, no larger (and no harder to manipulate) than a generic square function defined via convolutions with an arbitrary, smooth, compactly-supported kernel ψ. Using the intrinsic square function, we answer a question of R. Fefferman and E. M. Stein. If time permits, we will show some of the intrinsic square function's other nice features as well.