Exam 3 - (50 pts)
Stat 145 – Fall 2009

Name KEY

For credit show or explain all answers.

Selected z* values from Table C:

C	90%	95%	99%
z*	1.645	1.960	2.576

1. (3 points) Michelle has a bag of colored candy-coated chocolates. The probabilities of each color are:

Brown	Yellow	Red	Blue	Orange	Green
0.15	???	0.10	???	0.20	???
	.30				.15

The probability of drawing a brown or a green candy is 0.30, and the probability of not drawing a yellow candy is 0.70. What is the probability of drawing a blue candy?

+1 P(Green) = .30 − .15 = .15

+1 P(yellow) = 1 − .7 = .30

P(Blue) = 1 − (.15 + .30 + .10 + .20 + .15)

1 − .9

= .10

+1

2. Adam and Becky saved their favorite pieces of Halloween candy for last. Adam has a peanut butter cup, a chocolate bar, and a pack of gum in his sack. Becky has a peanut butter cup and a chocolate bar in her sack. One candy is drawn from Adam's sack and another candy is drawn from Becky's sack, and the candy sequence is recorded. Assume the selection of each candy in each sack is equally likely. A: P, C, G

B: P, C

(a) (3 pts) What is the sample space? In other words, write down all arrangements of items, using P to denote a peanut butter cup, C to denote a chocolate bar, and G to denote a pack of gum. For example, PC means the item drawn from Adam's sack was a peanut butter cup and the item drawn from Becky's sack was a chocolate bar.

+3 S = { PP, PC, CP, CC, GP, GC }

(b) (3 pts) What is the probability that the candies drawn from each sack are the same?

2/6 = 1/3 = .33

+3

3. (3 pts) Two special tetrahedral (four-sided) dice are rolled. On each die, each side is labeled with 1, 2, 3, or 4 dots. After each roll, the sum of the number of dots on the down-faced sides is recorded. Let E be the event that the sum is even. Let F be the event that the sum is 5 or more. Are E and F disjoint? Why or why not?

$$S = \{2, 3, \ldots, 7, 8\}$$
$$E = \{2, 4, 6, 8\}$$
$$F = \{5, 6, 7, 8\}$$

x3 E and F are not disjoint because they share the outcomes 6 + 8.

4. Studies of young surfers in Hawaii indicate that optimal levels of Vitamin D are approximately 20-60 ng/ml (nanograms/milliliter of serum). The vitamin D levels of surfers follow a Normal distribution with mean $\mu = 27$ ng/ml and standard deviation $\sigma = 17$ ng/ml.

(a) (3 pts) What is the probability that the vitamin D level of a randomly selected surfer is greater than 60 ng/ml?

$$z = \frac{x - \mu}{\sigma} = \frac{60 - 27}{17} = 1.94 \quad +1$$

$$1 - \underset{\text{T.A}}{.9738} \; {+1} = .0262 \; {+1}$$

* (b) (3 pts) What is the shape (a word), center (a number), and standard deviation (a number) of the sampling distribution? of n = 4 ?

+1 Shape: normal

+1 center: $\mu = 27$ ng/ml

+1 std. dev.: $\sigma/\sqrt{n} = 17/\sqrt{4} = 8.5$ ng/ml

(c) (3 pts) What is the probability that the mean vitamin D level of 4 randomly selected surfers is greater than 60 ng/ml?

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{60 - 27}{17/\sqrt{4}} = 3.88 \qquad P(\bar{x} > 60) \approx 0$$

+1 +1 +1

5. A manufacturer produces tin cans with wall thickness having a Normal distribution with a standard deviation of σ = .07 mm. Tin cans that are too thick or too thin are undesirable. Optimal can thickness is 0.53 mm. A sample of 25 cans is randomly selected and the sample mean is found to be 0.50 mm. Is this evidence that the mean can width differs from the optimal width?

(a) (2 pts) State your hypotheses using mathematical notation (symbols).

+1 $H_0: \mu = .53$ mm

+1 $H_a: \mu \neq .53$ mm

(b) (2 pts) In words, state what the null hypothesis means.

+2 The true mean can width is equal to .53 mm.

(c) (2 pts) Calculate the value of the test statistic. +1

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{.50 - .53}{.07/\sqrt{25}} = -2.14 \quad +1$$

(d) (2 pts) Determine the p-value.

T.A.

$2(.0162) = .0324$

(e) (2 pts) State your conclusion in terms of the problem. +1 +1

+2 There is strong evidence the true mean can width is not equal to .53 mm.

(f) (3 pts) Is the result above significant at the 1% level (α = 0.01)? Why or why not? +2

p-value α-level

.0324 > .01 → +1 NOT Significant because the p-value is greater than the α-level

or

"at the 1% level of significance there is no evidence the true mean width of the cans is different from .53 mm"

6. Suppose the distribution of the heights of 18-year old men in the United States is approximately Normal with unknown mean μ and known standard deviation $\sigma = 2.8$ inches. A simple random sample of size 36 from this population yields a sample mean of 69.4 inches.

(a) (3 pts) Give a 95% confidence interval for the mean height of 18-year old men in the United States.

CI: $\bar{x} \pm z^* \sigma/\sqrt{n}$ $\quad z^* = 1.96$ @ $C = 95\%$

+2 $69.4 \pm (1.96)(2.8/\sqrt{36})$

69.4 ± 0.9

+1 $(68.5, 70.3)$

(b) (2 pts) Interpret this interval.

+2 "We are 95% confident the true mean height of 18-yr-old men in the US. is between 68.5 and 70.3 inches."

(c) (2 pt) Without doing any further calculations, explain what would happen to the width of the interval above if the confidence level were changed to 90%.

+2 When the confidence level decreases, z^* decreases, and the margin of error (width of interval) decreases.

(d) (2 pts) How many 18-year old men must we sample if we want to estimate the mean height μ within a margin of error of ±0.4 inches with 95% confidence?

$$n = \left(\frac{z^* \sigma}{ME}\right)^2 = \left(\frac{(1.96)(2.8)}{.4}\right)^2 = 186.3 \rightarrow 187$$

+1 +1

(e) (1 pt) What is the critical value (z^*) for a confidence level of 97.5%?

$\frac{1-.975}{2} = .0125 \rightarrow$ T.A. "guts" $\rightarrow z^* = |-2.24|$

$z^* = 2.24$ +1

7. The distribution of the heights of adults in the United States is bimodal (i.e., has two peaks).

(a)(2pts) If a sample of 2000 adults were selected, would the sampling distribution of the sample mean be approximately Normal? Why or why not? +1

When n is large +1, the central limit theorem states that the distribution of the sample means is approximately normal regardless of the shape of the population distribution.

(b)(2pts) If the sample size were increased, would the sampling variability increase, decrease, or stay the same? Explain your answer. +1

+1 The standard deviation is a measure of variability. The std. dev. of the sampling distribution is $\sigma / \sqrt{n}$. Thus, as n increases, the variability decreases.

8. (2 pts) The mean salary of employees at Cost-U-Less is \$32,722. A random sample of 22 employees had a mean of \$31,963. Which number is a parameter and which number is a statistic? Explain your answer.

+1 \$32,722 — Parameter $\Rightarrow$ comes from all individuals in the population

+1 \$31,962 $\Rightarrow$ statistic $\rightarrow$ comes from the random sample of $n = 22$.