GOALS FOR MATH 316: INTRODUCTION TO DIFFERENTIAL EQUATIONS

1. The students should review the calculus by deepening their knowledge of the antiderivative, the product rule & integration by parts, the chain rule, and Taylor's Theorem.

2. The students should be able to apply the special solution techniques:

   (a) 1st order linear $y' + p(t)y = f(t)$ by variation of parameters

   (b) 1st order separable $y' = f(x)/g(y)$ by separation

   (c) Linear 2x2 system $x' = Ax + g(t)$. Here the homogeneous problem is solved by the eigen approach and the nonhomogeneous using variation of parameters

   (d) second order linear constant coefficient nonhomogeneous $y'' + py' + qy = g(t)$. Here the homogeneous problem is solved by the eigen approach and the nonhomogeneous using undetermined coefficients and variation of parameters. The student should understand forced vibrations, frequency response and resonance.

3. The students should understand that the Laplace transform is a basic tool of Applied Mathematics and understand it as a solution technique for linear ODEs

   (a) Know the definition of the LT and LT of derivative

   (b) Be able to use given Table, by first finding appropriate forms, eg using partial fraction expansions

   (c) Find the solution of $y'' + py' + qy = g(t)$ by LT

   (d) Be able to work with right hand sides $g(t)$ which are δ-functions and know the role of step functions in the solution

   (e) Apply Convolution Theorem and LT to solve the nonhomogeneous problem, as an alternative to the method of variation of parameters in 2(c,d)

4. Be able to find the general solution, that is, the set of all solutions, for linear ODEs.

5. The students, with regard to numerical methods for integrating ODE's, should:

   (a) Be able to integrate a system of ODE's using the state of the art integrators in Matlab

   (b) Understand the Euler and improved Euler methods and the idea behind a variable step size code using these two

6. The students should understand the following basic linear Algebra of 2x2 matrices

   (a) Solubility of $Ax = b$

   (b) The eigen problem $Ax = \lambda x$

7. The students should understand the following introductory ideas from the qualitative theory of ODE's:
   (a) For a first order non-autonomous DE $y' = f(t, y)$
      i. Plot direction fields for simple problems by hand
      ii. Plot direction fields using MATLAB
      iii. Given a direction field, sketch solution curves.
   (b) For a first order autonomous DE $y' = f(y)$
      i. Draw phase line, showing equilibrium solutions and their stability
      ii. Draw direction fields and solution curves
   (c) For second order systems $x' = F(x, y), y' = G(x, y)$
      i. Find equilibrium solutions
      ii. Linearize about equilibria and determine linear stability
      iii. Construct phase plane portrait using (i), (ii) and intuition or MATLAB.
      iv. Analyze the nonlinear conservative system $x'' + g(x) = 0$ using the energy method and the potential function

A GENERAL SYLLABUS FOR MATH 316, with reference to the current book

Texts: Brannan and Boyce(BB) **Differential Equations**, and Polking and Arnold(PA) **Ordinary Differential Equations using Matlab**, third edition.

Prerequisites: Math163 and CS151

Course Outline

- Weeks 1-2: First Order Equations $\frac{dy}{dx} = f(x, y)$
  Here the emphasis is on geometry, solution techniques and numerical approximations.
  - Direction fields, solutions and Euler's method
  - Linear $y' + p(x)y = g(x)$; integrating factor and variation of parameter
  - Separable equations $M(x) + N(y)\frac{dy}{dx} = 0$
  - Improved Euler and variable step methods
  - Phase line and stability for the autonomous case

  See Sections 1.1-1.3, 2.1-2, 2.5, 2.7-8 in BB and Chapters 3 and 5 in PA
  Optional material from other sections in Chapter 2 if time. Students without Matlab experience should see the instructor ASAP and consider taking or auditing CS151.

- Weeks 3-4: Systems of First Order DEs 2x2 case
  $x' = ax + by; y' = cx + dy$
  - Matrix formulation and elementary matrix manipulations
  - Eigen problem and general solution
  - Enough theory to argue that the general solution (i.e., the set of all solutions) is a LC of two LI solutions
  - Phase Plane
  - Fundamental Solution Matrix (briefly)
  - 4th order Runge Kutta

  Most of Chapter 3

- Week 5 Homogeneous Second Order Equations $ay'' + by'' + cy = 0$
  - Constant coefficient homogeneous

    - Enough theory to argue that the general solution in the homogeneous case is a linear combination of two LI solutions

  Sections 4.1-4.4

- Week 6: Catch up and Exam

- Week 7: Nonhomogeneous Second Order Equations $ay'' + by'' + cy = g(t)$

    - General solution = G.S. of homogeneous plus any particular solution
    - Method of Undertermined Coefficients
    - Variation of Parameters - Main point it always works, but more complicated than UC
    - Harmonic and forced harmonic motion

  Relevant Sections 4.5-4.8

- Weeks 8-9: Laplace Transform
  Chapter 5

- Weeks 10-11 More Systems

- Week 12: Catch up and Exam

- Weeks 13-15: Nonlinear Equations and Stability

    - Autonomous systems in the plane
      $x' = f(x, y); y' = g(x, y)$
        * Equilibrium solutions and stability
        * Linearization about equlibrium solutions
        * Phase plane portraits with emphasis on $x'' + g(x) = 0$ and the energy method
    - Observe chaotic trajectories in a specific 3D system