Stat 145: Exam 3 Review Answers

1. (a) The probability is:

$$\begin{aligned} P(X < 15) &= P\left(\frac{X - 31.8}{10} < \frac{15 - 31.8}{10}\right) \\ &= P(Z < -1.68) \\ &= .0465 \end{aligned}$$

   (b) The mean of the sampling distribution is 31.8 and the standard deviation is:

$$\frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{25}} = 2$$

   (c) Yes, the sampling distribution of $\bar{X}$ is approximately Normal because the population distribution is approximately Normal.

   (d) The probability is:

$$\begin{aligned} P(\bar{X} > 35) &= P\left(\frac{\bar{X} - 31.8}{2} > \frac{35 - 31.8}{2}\right) \\ &= P(Z > 1.6) \\ &= 1 - .9452 \\ &= .0548 \end{aligned}$$

2. (a) You must check two things:
   - Are all probabilities between 0 and 1? Yes.
   - Do the probabilities sum to 1? Yes.

$$0.000 + 0.003 + 0.060 + 0.062 + 0.036 + 0.121 + 0.691 + 0.027 = 1$$

   (b) The probability is $0.000 + 0.003 + 0.060 + 0.062 = 0.125$.

   (c) The probability is $1 - 0.691 = 0.309$.

3. The population is all adult residents in Albuquerque. The parameter is the percentage of adult residents in Albuquerque who favor the mayor's proposal. The sample is the 900 adult residents randomly chosen. The statistic is 40%.

4. Jill's argument is based on the Law of Large Numbers.

5. (a) The nine arrangements are: XX, XY, XZ, YX, YY, YZ, ZX, ZY, and ZZ.

(b) The outcomes that make up event A are: XX, YY, and ZZ.

(c) The probability is $(1/9) + (1/9) + (1/9) = 1/3$.

(d) The outcomes that make up event B are: XY, XZ, YX, YZ, ZX, and ZY.

(e) The probability is $(1/9) + (1/9) + (1/9) + (1/9) + (1/9) + (1/9) = 2/3$.

(f) They do not have any outcomes in common.

6. (a) The hypotheses are:

$$
\begin{aligned}
H_0 &: \mu = 98.6 \\
H_A &: \mu \neq 98.6
\end{aligned}
$$

The value of the test statistic is:

$$
\begin{aligned}
z &= \frac{98.4 - 98.6}{0.72/\sqrt{65}} \\
&= -2.24
\end{aligned}
$$

The P-value is:

$$
\begin{aligned}
P &= P(Z \leq -2.24) + P(Z \geq 2.24) \\
&= 2P(Z \leq -2.24) \\
&= 2(.0125) \\
&= .025
\end{aligned}
$$

We reject the null hypothesis and conclude that the mean female body temperature is different from 98.6 degrees Fahrenheit.

(b) A 95% confidence interval for μ is:

$$
\begin{aligned}
98.4 \pm 1.96\left(\frac{0.72}{\sqrt{65}}\right) &= 98.4 \pm 0.175 \\
&= (98.225, 98.575)
\end{aligned}
$$

(c) This method produces a confidence interval that contains μ in 95% of all samples.

(d) In this problem, the value z^* is the number that captures central probability 0.94 under a standard Normal curve between $-z^*$ and z^*. We want to find the value z^* with probability $0.94 + 0.03 = 0.97$ to the left. Using Table A, the closest probability to 0.97 is 0.9699. Thus, $z^* = 1.88$.

(e) The sample size should be:

$$\begin{aligned} n &= \left(\frac{(1.96)(0.72)}{0.1}\right)^2 \\ &\doteq 199.15 \end{aligned}$$

Rounding up to the next higher whole number, use a sample size of $n = 200$.

(f) Changing the confidence level to 90% would decrease the sample size, as the value of z^* for 90% confidence is smaller than the value of z^* for 95% confidence.

7. (a) The hypotheses are:

$$H_0 : \mu = 11$$
$$H_A : \mu > 11$$

The value of the test statistic is:

$$\begin{aligned} z &= \frac{11.4 - 11}{4/\sqrt{64}} \\ &= 0.80 \end{aligned}$$

The P-value is:

$$\begin{aligned} P &= P(Z \geq 0.80) \\ &= 1 - .7881 \\ &= .2119 \end{aligned}$$

We fail to reject the null hypothesis and conclude that the mean number of unoccupied seats per flight has not increased in the last year.

(b) The formula for the margin of error is:

$$m = z^* \left(\frac{\sigma}{\sqrt{n}}\right)$$

By substitution we get:

$$0.98 = z^* \left(\frac{4}{\sqrt{64}}\right)$$

Solving for z^* we get:

$$\begin{aligned} (0.98)(\sqrt{64}) &= 4z^* \\ \frac{(0.98)(\sqrt{64})}{4} &= z^* \\ 1.96 &= z^* \end{aligned}$$

If $z^* = 1.96$, then the confidence level was 95%.