

## Chapter 15 Solutions

**15.3** 98.8% and 22 are statistics; they are based on the survey of 500 American anabolic androgenic steroid users; 50.0% is a parameter (it is true for all Americans ages 16 to 62).

**15.6 (a)** The population is the 12,443 respondents to the American Time Use Survey; the population distribution (Normal with mean 528.8 minutes and standard deviation 137.2 minutes) describes the minutes of sleep per night for the *individuals* in this population. **(b)** The sampling distribution (Normal with mean 528.8 minutes and standard deviation 13.72 minutes) describes the distribution of the *average* sleep time for 100 randomly selected individuals from this population.

**Note:** *Students may say the population is all Americans. That is the population from which the individuals who responded to the survey were drawn, but because we are sampling again from those respondents, this becomes the population of interest.*

**15.9 (a)** The sampling distribution of  $\bar{x}$  is  $N(115, 25/\sqrt{100}) = N(115 \text{ mg/dl}, 2.5 \text{ mg/dl})$ . Therefore,  $P(112 < \bar{x} < 118) = P(-1.2 < Z < 1.2) = 0.8849 - 0.1151 = 0.7698$ , using Table A. **(b)** With  $n = 1000$ , the sample mean has the  $N(115 \text{ mg/dl}, 0.7906 \text{ mg/dl})$  distribution, so  $P(112 < x < 118) = P(-3.79 < Z < 3.79) = 0.9998$ .

**15.10 (a)**  $\sigma/\sqrt{n} = 10/\sqrt{4} = 5 \text{ mg}$ . **(b)** Solve  $\sigma/\sqrt{n} = 2$ , or  $10/\sqrt{n} = 2$ , so  $\sqrt{n} = 5$ , or

$n = 25$ . The average of several measurements is more likely than a single measurement to be close to the mean.

**15.11** No. The histogram of the sample values will look like the population distribution, whatever it might happen to be. (For example, if we roll a fair die many times, the histogram of sample values should look relatively flat—probability close to  $1/6$  for each value 1, 2, 3, 4, 5, and 6.) The central limit theorem says that the histogram of *sample means* (from many large samples) will look more and more Normal.

**15.13** STATE: We ask, what is the probability that the average loss for 10,000 such policies will be no greater than \$135, when the long-run average loss is \$125? PLAN: Use the central limit theorem to approximate this probability. SOLVE: The central limit theorem says that, in spite of the skewness of the population distribution, the average loss among 10,000 policies will be approximately  $N(\$125, \$300/\sqrt{10,000}) = N(\$125, \$3)$ . Now,  $P(\bar{x} \leq \$135) = P(Z \leq \frac{135-125}{3}) = P(Z \leq 3.33) = 0.9996$ .

CONCLUDE: We can be about 99.96% certain that average losses will not exceed \$135 per policy.

**15.17** (b) statistic; this is a proportion of the people interviewed in the sample of 60,000 households.

**15.18** (c) parameter; 61.1% is a proportion of all registered voters (the population).

**15.19** (b) The law of large numbers says that the mean from a large sample is close to the population mean. Statement (c) is also true, but is based on the central limit theorem, not on the law of large numbers.

**15.20** (a) The mean of the sample means ( $\bar{x}$  values) is the same as the population mean ( $\mu$ ).

**15.21** (c) The standard deviation of the distribution of  $\bar{x}$  is  $\sigma/\sqrt{n}$ .

**15.22** (a) "Unbiased" means that the estimator is right "on the average."

**15.23** (c) The central limit theorem says that the mean from a large sample has (approximately) a Normal distribution. Statement (a) is also true, but is based on the law of large numbers, not on the central limit theorem.

**15.24** (b) For  $n = 6$  women,  $\bar{x}$  has an  $N(266, 16/\sqrt{6}) = N(266, 6.5320)$  distribution, so  $P(\bar{x} > 270) = P(Z > 0.61) = 0.2709$ .

**15.27** In the long run, the gambler earns an average of 94.7 cents per bet. In other words, the gambler loses (and the house gains) an average of 5.3 cents for each \$1 bet.

**15.29** Let  $X$  be Shelia's measured glucose level. **(a)**  $P(X > 130) = P(Z > 0.67) = 0.2514$   
**(b)** If  $\bar{x}$  is the mean of four measurements (assumed to be independent), then  $\bar{x}$  has an  $N(122, 12/\sqrt{4}) = N(122 \text{ mg/dl}, 6 \text{ mg/dl})$  distribution, and  $P(\bar{x} > 130) = P(Z > 1.33) = 0.0918$ .

**15.31** As shown in Exercise 15.29(b), the mean of four measurements has an  $N(122 \text{ mg/dl}, 6 \text{ mg/dl})$  distribution, and  $P(Z > 1.645) = 0.05$  if  $Z$  is  $N(0,1)$ , so  $L = 122 + 1.645 \times 6 = 131.87 \text{ mg/dl}$ .

**15.35** STATE: What are the probabilities of an average return over 10%, or less than 5%? PLAN: Use the central limit theorem to approximate this probability. SOLVE: The central limit theorem says that over 40 years,  $\bar{x}$  (the mean return) is approximately Normal with mean  $\mu = 13.3\%$  and standard deviation  $17.0\%/\sqrt{40} = 2.688\%$ . Therefore,  $P(\bar{x} > 10\%) = P(Z > -1.23) = 0.8907$ , and  $P(\bar{x} < 5\%) = P(Z < -3.09) = 0.0010$ . CONCLUDE: There is about an 89% chance of getting average returns over 10%, and a 0.01% chance of getting average returns less than 5%.

**Note:** We have to assume that returns in separate years are independent.

