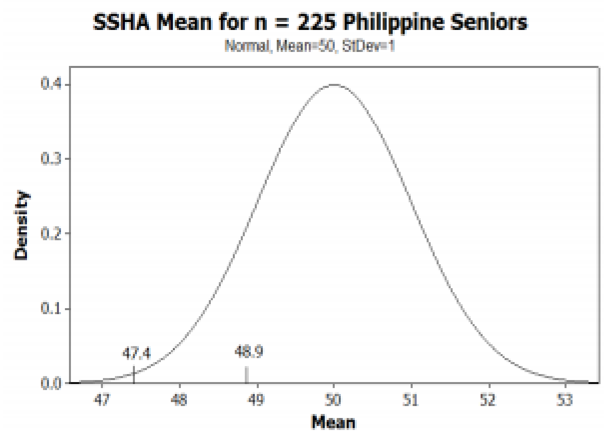
Chapter 17 – Tests of Significance: The Basics

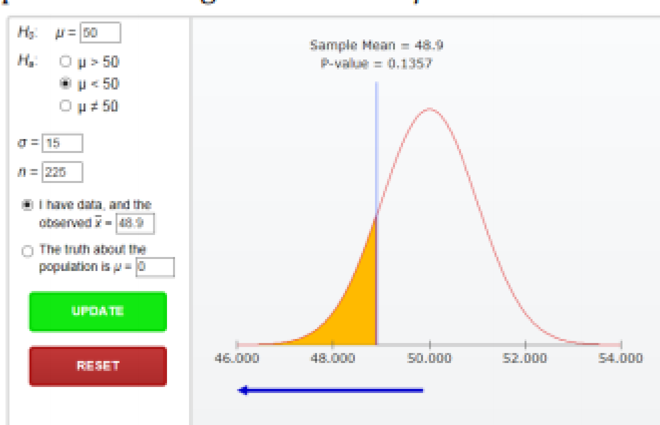
17.1 (a) If $\mu = 50$, the distribution is approximately Normal with mean $\mu = 50$ and standard deviation $\frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{225}} = 1$. **(b)** The actual result lies out toward the low tail of the curve, while 48.9 is fairly close to the middle. If $\mu = 50$, observing a value similar to 48.9 would not be too surprising, but 47.4 is much less likely, and it therefore provides some evidence that $\mu < 50$.

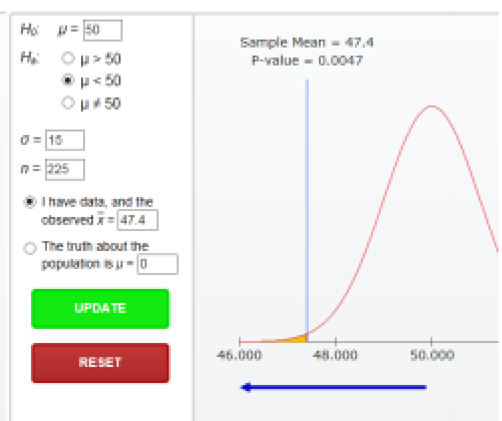


17.3 $H_0: \mu = 50$ vs. $H_a: \mu < 50$. Because the teacher suspects that poor attitudes are, in part, responsible for the decline in scores, we look to see if attitude scores are decreasing.

17.5 $H_0: \mu = 75$ vs. $H_a: \mu < 75$. The professor suspects this section's students perform worse than the population of all students in the class on average.

17.11 (a) The P-value for $\bar{x}$ = 48.7 is 0.1357. This is not significant at either $\alpha = 0.05$ or $\alpha = 0.01$. **(b)** The P-value for $\bar{x}$ = 47.4 is 0.0047. This is significant at both $\alpha = 0.05$ and $\alpha = 0.01$. **(c)** If $\mu = 50$ (that is, if H_0 were true), observing a value similar to 48.7 would not be too surprising, but 47.4 is much less likely, and it therefore provides strong evidence that $\mu < 50$.





17.14 STATE: Is there evidence that the true conductivity of the iron rod is not 10.1? PLAN: Let μ be the rod's true conductivity (the mean of all measurements of its conductivity). We test H_0: $\mu = 10.1$ against H_a: $\mu \neq 10.1$; we are concerned with deviations in either direction, so this is a two-sided alternative. SOLVE: Assume we have a Normal distribution and an SRS. We have $\bar{x} = 10.0833$, which is obtained from the data. The standard deviation of $\bar{x}$ is $\frac{0.1}{\sqrt{6}} = 0.0408$, so the test statistic is

$z = \frac{10.0833 - 10.1}{0.0408} = -0.41$. The P-value is $2P(Z \leq -0.41) = 0.6818$. CONCLUDE: This sample gives little reason to doubt that the true conductivity is 10.1. There is virtually no evidence that the true conductivity of the rod differs from 10.1. Random chance easily explains the observed sample mean.

17.17 This is not significant at the $\alpha = 0.05$ level because z is not larger than 1.96 or less than -1.96. It is also not significant at $\alpha = 0.01$ because $|z|$ is smaller than 2.576.

17.18 (a) $z = \frac{0.4365 - 0.5}{0.2887/\sqrt{100}} = -2.20$ **(b)** This result is significant at the 5% level because $z < -1.96$. **(c)** It is not significant at the 1% level because $z \geq -2.576$. **(d)** This value of z is between 2.054 and 2.326, so the P-value is between 0.02 and 0.04 (because the alternative is two-sided).

17.21 (a) This is the definition of a P-value.

17.22 (c) 0.008 is less than both 0.01 and 0.05.

17.23 (c) The P-value for $z = 2.41$ is 0.0080 (assuming that the difference is in the correct direction; that is, assuming that the alternative hypothesis was H_a $\mu > \mu_0$).

17.24 (b) $z = \frac{24.667 - 25}{1/\sqrt{3}} = -0.577$

17.25 (a) The null hypothesis states that μ takes on the "default" value, 18 seconds.

17.26 (b) The researcher believes that loud noises will make the rats complete the maze faster (decrease the completion time), so the alternative is one-sided.

17.27 (c) A small P-value means we should not (or should rarely) find an observed difference that is as large as or larger than what was seen in H_0 is true. The P-value does not tell us whether the difference seen is "large" or "practically important," nor does it refer to the probability that H_0 is true.

17.28 (c) This is a two-sided alternative, so we have 0.0025 in each tail of the Normal distribution, leading to $|z| > 2.807$.

17.29 (a) This is a one-sided alternative, so we have 0.005 in the right tail of the Normal distribution, leading to $z > 2.576$.

17.30 (a) H_0: $\mu = 15$ hours per week vs. H_a: $\mu > 15$ hours per week. **(b)** $z = \dfrac{15.3-15}{65/\sqrt{463}} = 0.10$ **(c)** P-value = $P(Z > 0.10) = 0.4602$; there is little evidence that the average amount of time spent studying by all students is more than 15 hours per week.