Chapter 21 – Comparing Two Means

21.1 This is a matched-pairs design. Each plot is a matched pair.

21.2 This involves two independent samples.

21.3 This involves a single sample.

21.4 This involves two independent samples (because the results for the new battery are independent of the results for the prototype battery).

21.6 STATE: Does the average time lying down differ between obese people and lean people? PLAN: We test $H_0: \mu_1 = \mu_2$ versus $H_a: \mu_1 \neq \mu_2$, where μ_1 is the mean time spent lying down for the lean group, and μ_2 is the mean time for the obese group. SOLVE: We assume that the data come from SRSs of the two populations. See Example 21.2 for a discussion of conditions for inference applied to this problem. The stemplots do not indicate non-Normal data. We proceed with the *t* test for two samples. With $\bar{x}_1$ = 501.6461, $\bar{x}_2$ = 491.7426, s_1 = 52.0449, s_2 = 46.5932, n_1 = 10, and n_2 = 10: SE = $\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$ = 22.0898 and $t = \frac{\bar{x}_1 - \bar{x}_2}{\text{SE}}$ = 0.448. Using df as the smaller of 10−1 and 10−1, we have df = 9, and *P* > 0.50. Using software, df = 17.8 and *P* = 0.6593. CONCLUDE: There is no evidence to support a conclusion that lean people spend a different amount of time lying down (on average) than obese people.

Lean		Obese
9	3	
	4	1
	4	
5	4	44
6	4	6
8	4	
10	5	011
33	5	23
5	5	
6	5	6

21.13 Here are the details of the calculations:

$$\text{SE}_{Alone} = \frac{0.68}{\sqrt{37}} = 0.1118$$

$$\text{SE}_{Friends} = \frac{0.83}{\sqrt{21}} = 0.1811$$

$$\text{SE} = \sqrt{SE_{Alone}^2 + SE_{Friends}^2} = 0.21283$$

$$\text{df} = \frac{SE^4}{\frac{1}{36}\left(\frac{0.68^2}{37}\right)^2 + \frac{1}{20}\left(\frac{0.83^2}{21}\right)^2} = \frac{0.00205}{0.00005815} = 35.284$$

$$t = \frac{0.29-(-0.19)}{0.21283} = 2.255$$

21.14 Let μ_1 denote the mean for men and μ_2 denote the mean for women. According to the output, $\bar{x}_1 = -19.50$, $\bar{x}_2 = -12.71$, $s_1 = 5.612$, and $s_2 = 5.589$. With n_1 = 6 and n_2 = 7:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{-19.50 - (-12.71)}{\sqrt{\frac{5.612^2}{6} + \frac{5.589^2}{7}}} = -2.179.$$

Also,

$$\text{df} = \frac{\left(\frac{5.612^2}{6} + \frac{5.589^2}{7}\right)^2}{\frac{1}{6-1}\left(\frac{5.612^2}{6}\right)^2 + \frac{1}{7-1}\left(\frac{5.589^2}{7}\right)^2} = 10.68\text{, rounded to two places.}$$

21.15 Reading from the software output shown in the statement of Exercise 21.13, we find that there is a significant difference in mean perceived formidability for men alone and with friends ($t = 2.255$, df = 35.3, $P < 0.02$). Because larger scores indicate greater perceived formidability, it appears that foes appear more formidable when alone as opposed to when with friends.

21.18 (c) a one-sample *t* interval. There is one sample, and only one score comes from each member of the sample.

21.19 (a) a two-sample *t* test. We have two independent populations: females and males.

21.20 (b) a matched-pairs *t* test. Two measurements (one for each device) are being taken on each person.

21.21 (b) Confidence levels and *P*-values from the *t* procedures are quite accurate even if the population distributions are not exactly Normal.

21.22 (c) 20. Here, df is the lesser of (21 – 1) and (21 – 1).

21.23 (b) $\frac{15.84 - 9.64}{\sqrt{\frac{8.65^2}{21} + \frac{3.43^2}{21}}} = 3.05$

21.27 (a) To test the belief that women talk more than men, we use a one-sided alternative. $H_0: \mu_F = \mu_M$ versus $H_a: \mu_F > \mu_M$. **(b)–(d)** The small table below provides a summary of *t* statistics, degrees of freedom, and *P*-values for both studies. The two sample *t* statistic is computed as $t = \frac{\bar{x}_F - \bar{x}_M}{\sqrt{\frac{s_F^2}{n_F} + \frac{s_M^2}{n_M}}}$, and we take the conservative approach for computing df as the smaller sample size, minus 1.

Study	*t*	df	Table C values	*P*-value
1	−0.248	55	$\lvert t \rvert < 0.679$	$P > 0.25$
2	1.507	19	$1.328 < t < 1.729$	$0.05 < P < 0.10$

Note that for Study 1 we reference df = 50 in Table C. **(e)** The first study gives no support to the belief that women talk more than men; the second study gives weak support, significant only at a relatively high significance level (say $\alpha = 0.10$).

21.29 (a) Call group 1 the Stress group, and group 2 the No stress group. Then, because SEM = $s/\sqrt{n}$, we have $s = \text{SEM}\sqrt{n}$. $s_1 = 3\sqrt{20} = 13.416$ and $s_2 = 2\sqrt{51} = 14.283$. **(b)** Using conservative Option 2, df = 19 (the lesser of 20 – 1 and 51 – 1). **(c)** We test $H_0: \mu_1 = \mu_2$ versus $H_a: \mu_1 \neq \mu_2$. With $n_1 = 20$ and $n_2 = 51$, SE = $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ =

3.605, and $t = \frac{\bar{x}_1 - \bar{x}_2}{\text{SE}} = \frac{26 - 32}{3.605}$ = −1.664. With df = 19, using Table C, $0.10 < P < 0.20$. There is little evidence in support of a conclusion that mean weights of rats in stressful environments differ from those of rats without stress.