ALGEBRA QUALIFYING EXAMINATION: JANUARY 2015

There are 10 problems. Each problem is worth 10 points. Please write your banner ID on the top of each page.

(1) Let G be a nonabelian group of order p^3. Show that $Z(G) = G'$.
(2) Show there is no simple group of order 192.
(3) Let R be a principal ideal domain and S a multiplicatively closed subset of R. Show that $S^{-1}R$ is a principal ideal domain.
(4) If R is a field show that all R-modules are projective and also injective.
(5) Let p be a prime. Construct a field with p^p elements.
(6) Give examples of field extensions over $\mathbb{Q}$ which have Galois group:
  (a) The Klein 4 group.
  (b) The cyclic group of order 4.
  (c) The dihedral group of order 8.

  Justify your answer.
(7) Prove that the ideal $(2, 1 + \sqrt{-5})$ in $\mathbb{Z}[\sqrt{-5}]$ is not principal.
(8) Prove that the polynomial $t^4 + t + 1$ is irreducible in $\mathbb{Q}[t]$.
(9) Prove that the group S_4 is solvable.
(10) Prove that the exact sequence of $\mathbb{Z}$-modules $0 \to \mathbb{Z} \to \mathbb{Q} \to \mathbb{Q}/\mathbb{Z} \to 0$ is not split.