ALGEBRA QUALIFIER AUGUST 2016

(1) Prove that if G and H are two finite groups with coprime orders then there is no non-trivial homomorphism between them.

(2) Let G be a group. Show that the set of inner automorphism on G is a normal subgroup of the group of automorphisms of G.

(3) Prove that the group of upper triangular $n \times n$ complex matrices is solvable.

(4) Prove that the ring $\mathbb{C}[x, y]/(y^2 - x^3)$ is not local.

(5) Prove that the ring $\mathbb{Z}[\sqrt{10}]$ is not a principal ideal domain.

(6) Prove that the polynomial $\sum_{i=0}^{8} t^i$ is reducible in $\mathbb{Q}[t]$

(7) Prove that there exist polynomials of degree 5 with coefficients in $\mathbb{Q}$ whose Galois group is S_5.

(8) Prove that if R is a ring and $e \in R$ satisfies $e^2 = e$ then $R/(e)$ is a projective R-module.

(9) Prove that if A and B are two complex diagonalizable commuting $n \times n$ matrices then there exists an invertible matrix U such that UAU^{-1} and UBU^{-1} are both diagonal.

(10) Consider the $\mathbb{C}[x]$-module

$$V = \frac{\mathbb{C}[x]}{(x-5)^7}.$$

Let $L : V \to V$ be the $\mathbb{C}$-linear map given by multiplication by x in V, and let A be the matrix of L with respect to some basis. Find the Jordan normal form of A.