

UNM Department of Mathematics and Statistics  
SYLLABUS FOR ALGEBRA QUALIFYING EXAM

Topics

- (1) Group Theory.
  - (a) Basic facts such as homomorphisms, subgroups, cosets, normal subgroups, quotients, automorphisms and short exact sequences.
  - (b) Symmetric and alternating groups.
  - (c) Free groups.
  - (d) Abelian groups and structure theorem for finitely generated abelian groups.
  - (e) Group actions.
  - (f) Sylow theorems.
  - (g) Nilpotent and solvable groups as well as subnormal and normal series.
  - (h) Krull Schmidt Theorem.
- (2) Ring Theory.
  - (a) Basic facts such as homomorphisms and subrings.
  - (b) Ideal Theory.
  - (c) Factorization in Commutative Rings: Euclidean Domains, PID's and UFD's.
  - (d) Polynomial Rings.
  - (e) Chain Conditions.
- (3) Module Theory.
  - (a) Basic facts such as homomorphisms, submodules, free modules, torsion modules and exact sequences.
  - (b) The structure theorem for finitely generated modules over PID's.
  - (c) Projective and injective modules.
  - (d) Hom and tensor products.
  - (e) Noetherian modules and chain conditions.
- (4) Linear Algebra.
  - (a) Basic facts such as vector spaces, linear transformations, linear independence, basis, dimension and determinants.
  - (b) Characteristic polynomials, eigenvalues and eigenvectors.
  - (c) Canonical forms.
- (5) Field Theory.
  - (a) Algebraic and transcendental extensions, splitting fields, algebraic closure, normality and separability.
  - (b) Galois theory.
  - (c) Finite fields.
  - (d) Cyclic, cyclotomic, abelian and solvable extensions.

Suggested References

- (1) Algebra by Hungerford, Springer-Verlag.
- (2) Algebra by Lang, Addison-Wesley.
- (3) Abstract Algebra by Dummit and Foote, John Wiley and Sons.