

## TOPICS for the NUMERICAL ANALYSIS QUALIFYING EXAM

The focus of the NA Qualifying Exam is Applied and Numerical Linear Algebra. It covers any of the topics listed below. The courses that help prepare for the exam are Math 464/514 and Math 504. These courses will address most (but not necessarily all) of the topics listed here.

### 1. Vector Spaces and inner product spaces.

- (a) Vector spaces, subspaces, linear independence, basis, dimension
- (b) Norms, basic inequalities
- (c) Inner product spaces, norms and orthogonality
- (d) Linear transformations and matrix representation
- (e) Change of basis and similar matrices

### 2. Linear Systems

- (a) Formulation using matrices
- (b) Geometric aspects of matrices: nullspace, range, solvability conditions, the four fundamental subspaces
- (c) Determinants (relation to volumes, product formula)
- (d) Overdetermined systems, least squares solutions, normal equations, generalized pseudoinverse
- (e) Rectangular systems, Row Echelon form, general solution to  $Ax=b$
- (f) Conditioning
- (g) Finite difference approximations to simple ODEs (eg 2-pt boundary value problems) and PDEs (eg heat or Poisson equation), error and order of convergence

### 3. Special matrices

- (a) Hermitian, unitary, and normal matrices; similar matrices; projectors; nilpotent matrices; positive definite matrices; inverses and generalized inverse (pseudoinverse)
- (b) Rank 1 matrices; low-rank perturbations of matrices; Sherman-Morrison-Woodbury formulas
- (c) Sparse matrices
  - storage and elimination for banded and almost-banded matrices;
  - sparsity in corresponding L,U factors
  - multiplication by sparse matrices
- (d) Fourier Transform
  - Discrete Fourier Transform
  - Properties of the FFT and applications to circulant and Toeplitz matrices

### 4. Matrix factorizations and numerical algorithms (see also 5e)

- (a) Concept of algorithm stability and backward error analysis
- (b) LU factorization
  - existence and use
  - Gauss Elimination algorithm, computational complexity and stability
  - Cholesky factorization, computational complexity and stability
- (c) QR factorization
  - existence and use
  - Gram-Schmidt algorithm, computational complexity and stability
  - algorithms using Householder reflections and Givens rotations, computational complexity and stability
- (d) Singular value decomposition
  - existence and use
  - construction
- (e) Applications to solve the least squares problem

## 5. Iterative methods for systems

- (a) Convergence of iterative processes
- (b) Classical matrix splitting methods - Jacobi, Gauss-Seidel, SOR, SSOR
- (c) Steepest descent algorithm
- (c) Krylov subspace methods:
  - conjugate gradient method for symmetric matrices
  - GMRES for nonsymmetric matrices

## 6. Eigenvalue problem

- (a) Existence of eigenvalues
- (b) Factorizations:
  - diagonalization, Jordan form, upper triangulation (Schur factorization)
  - applications to compute matrix functions (powers, exponential) and to solve ODEs
- (c) Conditioning
- (d) Numerical algorithms to compute eigenvalues
  - Rayleigh quotient, power method, inverse iteration with shifts
  - QR algorithm
  - Krylov subspace algorithm

### Recommended references:

- [1] C. D. Meyer, *Matrix analysis and applied linear algebra*, SIAM, 2000.
- [2] G. Strang, *Linear Algebra and its applications*, Harcourt Brace Jovanovich College Publ., 1988 (3rd ed).
- [3] L. N. Trefethen and D. Bau, *Applied Numerical Linear Algebra*, SIAM, 1997.
- [4] J. W. Demmel, *Applied Numerical Linear Algebra*, SIAM, 1997.
- [5] G. H. Golub and C. F. van Loan, *Matrix Computations*, Johns Hopkins University Press, 1996 (3rd ed).
- [6] A. Greenbaum, *Iterative methods for solving linear systems*, SIAM, 1997.
- [7] A. J. Laub, *Matrix Analysis for Scientists and Engineers*, SIAM, 2005.
- [8] P. Lax, *Linear Algebra*, Wiley, 1996.