In-class Examination

Ph. D. Qualifying and Masters Examination

January 11, 1999

1) Let

$$f_\theta(x) = \frac{1}{\pi}\frac{1}{1+(x-\theta)^2}, \qquad -\infty < x < \infty.$$

Consider testing $H_0 : \theta = 0$ versus $H_A : \theta = 2$ using one observation. Let Λ denote the likelihood ratio

$$\Lambda = \frac{\mathcal{L}(H_0)}{\mathcal{L}(H_A)} = \frac{f_0(x)}{f_2(x)}$$

where $f_2(x)$ indicates the density under $H_A : \theta = 2$ and similarly for $f_0(x)$.

(a) Obtain the critical region $\Lambda < 1$ as an equivalent region of x-values and determine α, the size of the Type I error.

(b) Is the test of $\Lambda < 1$ most powerful? Explain.

(c) Is the test of $\Lambda < 1$ uniformly most powerful for $H_0 : \theta = 0$ versus $H_A : \theta > 0$? Explain.

(d) Does the moment generating function of $f_\theta(x)$ exist? Why or why not?

2) Suppose X is a random variable such that $\mathrm{E}(X^4) < \infty$. Let $\mu = \mathrm{E}(X)$. Prove that

$$\Pr\left[|X-\mu| \geq \epsilon\right] \leq \frac{\mathrm{E}(X-\mu)^4}{\epsilon^4}, \quad \text{for } \epsilon > 0.$$

3) Let $Y_1, Y_2, \ldots, Y_n$ be a random sample with density

$$f_\theta(y) = \begin{cases} e^{-(y-\theta)} & y > \theta \\ 0 & \text{elsewhere} \end{cases}$$

where $\theta > 0$.

(a) Find the density of $Y_{(1)} \equiv \min\{Y_1, Y_2, \ldots, Y_n\}$.

(b) Is the density in (a) an exponential family? Explain.

(c) Show that $Y_{(1)}$ is a sufficient statistic for θ.

(d) Is $Y_{(1)}$ is a minimal sufficient statistic for θ? Explain.

(e) Find a function of $Y_{(1)}$ that is unbiased for θ.

(f) Find a estimator $\hat{\theta}_1$ for θ by the method of moments. If possible, adjust this estimator to make it unbiased.

(g) Find a estimator $\hat{\theta}_2$ for θ by the method of maximum likelihood. If possible, adjust this estimator to make it unbiased.

4) Suppose that the random variable Y hs a binomial distribution with n trials and success probability X, where n is a given constant and X is a Uniform(0,1) random variable.

(a) Find E(Y).

(b) Find the variance of Y.

(c) Find the joint distribution of X and Y. List limits on both X and Y for full credit.

(d) Find the marginal distribution of Y. (Hint: Recall that the beta function is

$$B(\alpha, \beta) = \int_0^1 u^{\alpha-1}(1-u)^{\beta-1} = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}.)$$

Take-home Examaination
Ph. D. Qualifying and Masters Examination
January, 1999

The examination is due at noon on Thursday, January 14, 1999. Exams should be given to Ronald Christensen - or any other statistics faculty member if he is not available.

1) Data on 60 men for the following variables are considered: y, weight in pounds; x_1, age in years; x_2, systolic blood pressure in millimeters of mercury; x_3, diastolic blood pressure in millimeters of mercury; x_4, cholesterol in milligrams per dl; x_5, height in inches. The data are accessible from the department network by using

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cp ~/fletcher/test.dat filename
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The file has an a case index as the first column, then the x variables in order, and the last column is y. Use the other variables to predict weight. Do a complete analysis. Summarize your results in no more than three typed, doublespaced pages. You may also include appendices that will be examined at the discretion of the grader.

2) Data are on the thickness of gold deposits on watch bezels as measured in microns. Factors involved in the electroplating are the amount of current, the pH value of the bath in which the current is applied, and the specific gravity of the the bath. The data are actually the average of six bezels, one from each rack. The factors are summaried in the first table. The data are given in the second table. Do a complete analysis of the data. Summarize your results in no more than three typed, doublespaced pages. You may also include appendices that will be examined at the discretion of the grader.

		Level		
Factor	Code	1	2	3
Current	A	11	13	15
pH	B	3.80 ± 0.02	3.90 ± 0.02	4.0 ± 0.02
Specific Gravity	C	1.09	1.105	1.12

A	B	C	$\bar{y}$
11	3.8	1.09	1.766
11	3.8	1.105	2.223
11	3.8	1.12	2.988
11	3.9	1.09	1.968
11	3.9	1.105	1.858
11	3.9	1.12	3.226
11	4.0	1.09	2.888
11	4.0	1.105	2.226
11	4.0	1.12	1.970
13	3.8	1.09	2.063
13	3.8	1.105	1.565
13	3.8	1.12	3.003
13	3.9	1.09	1.880
13	3.9	1.105	1.900
13	3.9	1.12	2.710
13	4.0	1.09	2.937
13	4.0	1.105	2.279
13	4.0	1.12	2.404
15	3.8	1.09	1.502
15	3.8	1.105	1.290
15	3.8	1.12	2.603
15	3.9	1.09	1.741
15	3.9	1.105	1.973
15	3.9	1.12	2.574
15	4.0	1.09	2.877
15	4.0	1.105	1.695
15	4.0	1.12	2.894