STATISTICS QUALIFYING EXAM

in-class part

August, 2002

General Instructions: Write your ID number on your report. Do not put your name on any of your answer sheets.

1. An investor has 21 thousand dollars to invest among 3 possible investments. Not all the money need be invested. Each investment must be in units of a thousand dollars. How many different investment strategies are possible?

2. Suppose X_1 and X_2 are independent discrete random variables with probability function

$$P(X_i = x) = x/6 \text{ for } x = 1, 2, 3$$

and zero otherwise.

   (a) Find the moment generating function (mgf) of X.
   (b) Find the mgf of $Y = X_1 + X_2$.
   (c) Use the mgf of Y to evaluate $E(Y)$.

3. Let $X_1, \ldots, X_n$ be *iid* with distribution

$$P(X \leq x|\alpha, \beta) = \begin{cases} 0 & \text{if } x < 0 \\ (x/\beta)^\alpha & \text{if } 0 \leq x \leq \beta \\ 1 & \text{if } x > \beta. \end{cases}$$

Here the parameters α and β are positive. Find the MLE's of α and β.

4. Let $X_1, \ldots, X_n$ be iid according to uniform distribution on $[-\theta, \theta]$. Find, if one exists, a best unbiased estimate of θ.

5. Let X be one observation from the distribution with density $f(x|\theta) = \pi^{-1}(1 + (x - \theta)^2)^{-1}$, $0 < x < \infty$. Show that the test

$$\phi(X) = \begin{cases} 1 & \text{if } 1 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

is most powerful of its size for testing $H_0 : \theta = 0$ versus $H_1 : \theta = 1$. Calculate Type I and Type II Error probabilities.

6. Let $X_1, \ldots, X_n$ be iid according to pdf $f(x|\theta) = \theta \exp(-\theta x)$, $x \geq 0$, $\theta > 0$. Find UMA $1 - \alpha$ confidence interval for θ.

7. Suppose $Y_i \sim$ independent Poisson(μ_i) for $i = 1, 2, \ldots, k$ where $\mu_i = \beta N_i$. Here β is unknown and $N_1, N_2, \ldots, N_k$ are fixed known constants.

   (a) Use the factorization theorem to obtain a sufficient statistic for β.

   (b) Compute the maximum likelihood estimate (MLE) $\hat{\beta}$ of β.

   (c) Show that $\hat{\beta}$ is unbiased for β and derive $var(\hat{\beta})$.

   (d) An alternative unbiased estimate is $\beta^* = \sum_{i=1}^{k}(Y_i/N_i)$. Compute $var(\beta^*)$.

   (e) Which estimator ($\hat{\beta}$ or β^*) is preferred? Discuss.