STATISTICS MASTER'S/PH.D. QUALIFYING EXAM, IN CLASS PORTION

January 13, 2003

This is a closed book, closed notes exam.

1. Let the continuous random vector (X, Y) be defined by the pdf $f_{X,Y}(x, y) = xe^{-x}e^{-y}$ on $x > 0$, $y > 0$. Define $U = X + Y$ and $V = X$. Find the marginal distribution of U.

2. Let X be the number of heads that occur in 3 flips of a coin for which p is the probability of obtaining a head on each flip, $0 < p < 1$. Given $X = x \in \{0, 1, 2, 3\}$, consider a second experiment consisting of flipping the same coin until an additional $(x+1)$ heads occur. Let Y be the number of tails that occur in *this second experiment* before we observe the additional $(x + 1)$ heads.

   (a) Find the form of the joint probability function of X and Y.

   (b) Evaluate $E(Y)$. Hint: First consider $E(Y|X = x)$.

3. Let $X_1, \ldots, X_n$ be *iid* with pdf $f(x|\theta) = \theta x^{\theta-1}$ on $0 \le x \le 1$, $0 < \theta < \infty$.

   (a) Find the moment generating function for the random variable $Y = \log X_1$.

   (b) Find the method of moments estimator (MOM) and maximum likelihood estimator (MLE) of θ.

   (c) Find the mean squared errors (MSE) of each of the estimators. Based on the MSE, which estimator is to be preferred?

4. Suppose that $X_1, \ldots, X_n$ are *iid Poisson*(λ). Find the best unbiased estimate of $\lambda \exp(-\lambda)$. Hint: Look at $P(X = 1)$.

5. Let $X_1, \ldots, X_n$ be iid Normal$(\theta, 1)$. Find a UMP test of size α for testing $H_0 : \theta \le \theta_0$ versus $H_1 : \theta > \theta_0$.

6. Suppose that we flip a coin 25 times and observe 17 heads. Let p be the probability of observing a head on a given toss. An exact (Binomial) test of the hypothesis $H_0 : p = 0.5$ against $H_1 : p > 0.5$ yields a p-value of 0.054.

   (a) In general, describe what a p-value measures and how it is typically used in an hypothesis testing setting.

   (b) What might you conclude from the p-value in the coin flipping experiment described above?

7. Let $X_1, X_2 \overset{iid}{\sim} exp(1)$. Find the distribution of the sample range $R = X_{(2)} - X_{(1)}$. Here, $X_{(1)} = \min\{X_1, X_2\}$ and $X_{(2)} = \max\{X_1, X_2\}$