STATISTICS MASTER'S/PH.D. QUALIFYING EXAM
TAKE-HOME PORTION: January 13, 2003

DIRECTIONS: Turn in your answers by noon on Friday, January 17, 2003, to the main office or to the proctor of the exam. Answer each question in two or less typed pages; you are encouraged to create a brief, well-organized appendix for each answer. You are to work on the exam independently.

1. An experiment was conducted on the effect of premixing speed and finish mixer speed on the center heights of cakes. Three different levels of speed were chosen for each of the two variables; increasing speed *level* indicates increasing mixing speed. Two cakes were baked at each level of premixing and finish mixing speed. The data are as follows:

Premix speed	Finish mix speed	Center heights
1	1	24, 17
1	2	23, 22
1	3	19, 15
2	1	23, 30
2	2	22, 22
2	3	20, 20
3	1	19, 10
3	2	21, 22
3	3	27, 29

Analyze this data completely keeping in mind the implied goal of the experiment. State and check assumptions. Be thorough.

2. The data set described below gives the percentage of body fat (B) determined by underwater weighing and various body circumference measurements for 252 men. This percentage (B) is calculated from using a method introduced by Siri (1956). We wish to build a regression model to predict the percentage of body fat (B) from age and the body measurements (excluding the density (D) from underwater weighing) because accurate measurement of body fat is inconvenient and costly. If the model gives accurate predictions, then we can use the fitted relationship to easily estimate the percent body fat.

Do an analysis with the stated goal in mind, and provide a measure of how accurately your model predicts body fat.

The data set is on the web at `http://www.stat.unm.edu/~hanson/bodyfat.txt`.

The variables listed in the data set, from left column to right, are:

- (D) Density determined from underwater weighing – *do not use in your analysis!*
- (B) Percent body fat from Siri's (1956) equation
- Age (years)
- Weight (*lbs*)
- Height (inches)
- Neck circumference (*cm*)
- Chest circumference (*cm*)
- Abdomen 2 circumference (*cm*)
- Hip circumference (*cm*)
- Thigh circumference (*cm*)
- Knee circumference (*cm*)
- Ankle circumference (*cm*)
- Biceps (extended) circumference (*cm*)
- Forearm circumference (*cm*)
- Wrist circumference (*cm*)

Measurement standards are apparently those listed in Benhke and Wilmore (1974), pp. 45-48 where, for instance, the abdomen 2 circumference is measured "laterally, at the level of the iliac crests, and anteriorly, at the umbilicus." The data were generously supplied by Dr. A. Garth Fisher who gave permission to freely distribute the data and use for non-commercial purposes.

More Details on the data:

This information is provided for your interest but *may* not necessarily be pertinent to the analysis.

A variety of popular health books suggest that the readers assess their health, at least in part, by estimating their percentage of body fat. In Bailey (1994), for instance, the reader can estimate body fat from tables using their age and various skin-fold measurements obtained by using a caliper. Other texts give predictive equations for body fat using body circumference measurements (e.g. abdominal circumference) and/or skin-fold measurements. See, for instance, Behnke and Wilmore (1974), pp. 66-67; Wilmore (1976), p. 247; or Katch and McArdle (1977), pp. 120-132).

Percentage of body fat for an individual can be estimated once body density has been determined. Folks (e.g. Siri (1956)) assume that the body consists of two components - lean body tissue and fat tissue. Letting

$$
\begin{aligned}
D &= \text{Body Density } (gm/cm^3) \\
A &= \text{proportion of lean body tissue} \\
B &= \text{proportion of fat tissue } (A + B = 1) \\
a &= \text{density of lean body tissue } (gm/cm^3) \\
b &= \text{density of fat tissue } (gm/cm^3)
\end{aligned}
$$

we have

$$D = 1/[(A/a) + (B/b)]$$

solving for B we find

$$B = (1/D)[ab/(a - b)] - [b/(a - b)].$$

Using the estimates a=1.10 gm/cm^3 and b=0.90 gm/cm^3 (see Katch and McArdle (1977), p. 111 or Wilmore (1976), p. 123) we come up with "Siri's equation":

$$\text{Percentage of Body Fat (i.e. } 100B) = 495/D - 450.$$

Volume, and hence body density, can be accurately measured a variety of ways. The technique of underwater weighing "computes body volume as the difference between body weight measured in air and weight measured during water submersion. In other words, body volume is equal to the loss of weight in water with the appropriate temperature correction for the water's density" (Katch and McArdle (1977), p. 113). Using this technique,

$$\text{Body Density} = WA/[(WA - WW)/c.f. - LV]$$

where

$$
\begin{aligned}
WA &= \text{Weight in air } (kg) \\
WW &= \text{Weight in water } (kg) \\
c.f. &= \text{Water correction factor} \\
LV &= \text{Residual Lung Volume (liters)}
\end{aligned}
$$

The water correction factor is equal to 1 at 39.2 degrees F as one-gram of water occupies exactly one cm^3 at this temperature, equal to 0.997 at 76-78 degrees F (Katch and McArdle (1977), p. 115). Other methods of determining body volume are given in Behnke and Wilmore (1974), p. 22.

References:

- Bailey, Covert (1994). Smart Exercise: Burning Fat, Getting Fit, Houghton-Mifflin Co., Boston, pp. 179-186.

- Behnke, A.R. and Wilmore, J.H. (1974). Evaluation and Regulation of Body Build and Composition, Prentice-Hall, Englewood Cliffs, N.J.

- Siri, W.E. (1956), "Gross composition of the body", in Advances in Biological and Medical Physics, vol. IV, edited by J.H. Lawrence and C.A. Tobias, Academic Press, Inc., New York.

- Katch, Frank and McArdle, William (1977). Nutrition, Weight Control, and Exercise, Houghton Mifflin Co., Boston.

- Wilmore, Jack (1976). Athletic Training and Physical Fitness: Physiological Principles of the Conditioning Process, Allyn and Bacon, Inc., Boston.