STATISTICS Ph. D. COMPREHENSIVE EXAM
August, 2003

General Instructions: Write your ID number on your answer sheets. Do not put your name on any of your answer sheets. All solutions should be rigorously explained. Show all work. If you cannot solve a problem, at least explain what is this problem about and what approach may lead to its solution.

Reminder: All assertions should be rigorously proved.

1. Let $X = (X_1, \dots, X_m)$ and $Y = (Y_1, \dots, Y_n)$ be samples from the normal distributions $N(\xi, \sigma^2)$ and $N(\eta, \tau^2)$. All these parameters are unknown. Find UMP unbiased acceptance region for $H: \ \tau^2/\sigma^2 = \delta_0$ versus $K: \ \tau^2/\sigma^2 \neq \delta_0$. What is the corresponding confidence set?

2. Prove the fundamental Neyman–Pearson Lemma (feel free to mention and prove: existence, sufficiency and necessity).

3. Let X be binomial $B(n,p)$. Under squared error loss, find the minimax estimate of p. Is it identical to the maximum likelihood estimate? If not, then compare risks of these estimates.

4. The underlying idea of MLE is based on the relation:

$$P_{\theta_0}\left(\prod_{l=1}^{n} f(X_l|\theta_0) > \prod_{l=1}^{n} f(X_l|\theta)\right) \to 1, \quad n \to \infty$$

whenever $\theta_0 \neq \theta$. Prove it (please do not forget to formulate your assumptions).