

Statistics Master and PH.D. Qualifying Exam
In Class: Monday, Jan 11 2010

Instructions: The exam has 6 multi-part problems. All of the problems will be graded. Write your ID number on your answer sheets. Do not put your name on any of the sheets. Be clear, concise, and complete. All solutions should be rigorously explained.

Problem 1. A company provides earthquake insurance. The premium X is modeled by the probability density function (p.d.f.)

$$f_X(x) = \frac{x}{5^2} e^{-x/5}, \quad 0 < x < +\infty$$

while the claims Y have the p.d.f.

$$g_Y(y) = \frac{1}{5} e^{-y/5}, \quad 0 < y < +\infty$$

If X and Y are independent, find the p.d.f. of $Z = X/Y$.

Problem 2. For the hierarchical model

$$Y|\Lambda \sim \text{Poisson}(\Lambda) \quad \text{and} \quad \Lambda \sim \text{Gamma}(\alpha, \beta)$$

find the marginal distribution, mean and variance of Y . Show that the marginal distribution of Y is a negative binomial if α is an integer.

Problem 3. Let X be a random variable with probability density function (p.d.f.)

$$f_X(x) = 2x e^{-x^2} I_{(0,+\infty)}(x)$$

- (a) Find the distribution of $Y = X^2$ by computing its moment generating function.
- (b) Let Y_1, \dots, Y_n be a random sample where the Y_i 's are identically distributed as Y . Find the distribution of $Z = \sum_{i=1}^n Y_i$.

Problem 4. Let X_1, X_2, \dots, X_n be a random sample with densities given by the p.d.f.

$$f(x) = \frac{1}{\theta} \exp\left(-\frac{x-\alpha}{\theta}\right) I_{[\alpha, \infty)}(x)$$

for $\theta > 0$ and $-\infty < \alpha < \infty$. Suppose θ and α are unknown.

- (a) Find a pair of statistics that are sufficient for (θ, α) .