Statistics Qualifying Exam: In Class Portion

Due on 2012-01-12

Please complete the problems on a seperate sheet of paper with your name at the top. Make sure to show your work and/or provide an explanation for each problem. Partial credit will be given when merited. *Note: This assignment may seem long, but we will have only four homeworks sets this quarter.*

Problem 1

Assume that we have observed a random sample $x_1, x_2, \ldots, x_n$ from a $N(\mu, 1)$ population. We are interested in testing the Hypotheses $H_0 : \mu = 0$ vs $H_1 : \mu \neq 0$. Assume that $n = 30$ and we have observed $\bar{x} = 1.35$.

(a) What is the form of the UMPU test with a fixed $\alpha = .05$?

(b) What is the p-value for this test?

(c) To conduct a Bayesian version of this test assume the following mixture prior for μ.

$$\pi(\mu) = \begin{cases} .5 & if \mu = 0 \\ .5 * \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\mu^2\right] & if \mu \neq 0 \end{cases}$$

What is the posterior probability that μ is equal to zero?

(d) Compare and contrast your answers in parts (b) and (c).

Problem 2

Assume that a random variable X has the following cumulative density function (cdf) given θ:

$$F_X(x|\theta) = \begin{cases} 0 & \text{if } x < 1 \\ \frac{\log(x)}{\log(\theta)} & \text{if } 1 \leq x < \theta \\ 1 & \text{if } \theta \leq x \end{cases}$$

where $\theta \in \{2, 3, \ldots\}$ and $\log(x)$ denotes the natural logirithm of x. Define the following two random variables:

$$Y = \lfloor X \rfloor = \text{the integer part of X}$$

$$W =< X >= \text{the fractional part of X.}$$

For example, $\lfloor 1.5 \rfloor = 1$, $< 1.5 >= .5$.

(a) Find the CDF of W, $F_W(w)$.

(b) Find the PDF of W, $f_W(w)$.

(c) Find the mean of W.

(d) Find the joint distribution of Y and W.

Problem 3

Let the random variable X have the following CDF:

$$F_X(x|\alpha,\beta) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x^\beta}{\alpha^\beta + x^\beta} & \text{if } x \geq 0. \end{cases}$$

Where $\alpha > 0$ and $\beta > 1$.

(a) Verify that F is a valid CDF.

(b) Find the density, $f_X(x|\alpha,\beta)$, of X.

(c) Find the median of X?

(d) Find $\mathrm{E}\left[X^k\right]$ for $k < \beta$.

(e) Find the distribution of $Y = \log\left[\left(\frac{X}{\alpha}\right)^\beta\right]$.

(f) Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n from F. Find the limiting distribution of $n^{1/\beta}X_{(1)}$.

(g) Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n from F. Find the limiting distribution of $\bar{X}$ when $\beta > 3$. If you did not answer (b), write the limiting distribution in terms of the mean and variance of X.