In-Class Statistics Masters and Ph.D. Qualifying Exam

August, 2015

Instructions: *The exam has 6 multi-part problems. All of the problems will be graded. Write your ID number on each of your answer sheets. Do not put your name on any of the sheets. Be clear, concise, and complete. All solutions should be rigorously explained.*

Problem 1. (20 pts) X is uniformly distributed on the interval $(0, 1)$, and Y is uniformly distributed on the interval $(0, 2)$. X and Y are independent.

$$U = XY \text{ and } V = \frac{X}{Y}.$$

Find the joint and marginal densities for U and V.

Problem 2. (15 pts) Box A has 2 red and 3 green balls. Box B has 4 red and 5 green balls. A ball is randomly selected from box A and placed in box B. With replacement after each draw, 5 balls are then randomly sampled from box B. Let X denote the number of red balls in this sample of 5 balls. Find the moment generating function of X.

Problem 3. (15 pts) X has a density $f_X(x) = \begin{cases} \alpha e^{-\alpha x}, & x > 0 \\ 0 & \text{elsewhere.} \end{cases}$ Y has a density $f_Y(y) = \begin{cases} \beta e^{-\beta y}, & y > 0 \\ 0 & \text{elsewhere.} \end{cases}$ X and Y are independent. Let $W = \text{maximum}(X, Y) - \text{minimum}(X, Y)$. Find the distribution function and pdf of W.

Problem 4. (20 pts) Let $X_1, X_2, \ldots, X_n$ equal the number of defects in n bolts of cloth. In particular $X_1, X_2, \ldots, X_n$ are iid with a Poisson(θ) distribution. Let $\tau(\theta)$ equal the probability that the number of defects in a particular bolt of cloth is at least 2.

(a) Find an unbiased estimator of $\tau(\theta)$.

(b) Construct a large sample confidence interval for $\tau(\theta)$.

(c) Find the UMVUE for $\tau(\theta)$.

(d) For a sample size of $n = 5$, construct the likelihood ratio test for $H_0 : \theta = \theta_0$ versus the alternative $H_1 : \theta \neq \theta_0$.

Problem 5. (10 pts)

Find the Neyman Pearson most powerful test of size α. Let $n = 1$, and, under H_0, let X have density:

$$f_0(x) = \begin{cases} 4x & 0 < x < \frac{1}{2} \\ 4 - 4x & \frac{1}{2} \leq x < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Under H_1, let X have density $f_1(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{elsewhere.} \end{cases}$

Problem 6. (20 pts) Let $X_1, X_2, \ldots, X_n$ be the survival times of n patients with malignant melanoma. Assume the survival times have the density

$$f(x) = \begin{cases} \dfrac{e^{-\frac{x}{\theta}}}{\theta} & x > 0 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Find the maximum likelihood estimate of $g(\theta) = P_\theta(X > t)$ for a fixed $t > 0$.

(b) Find the maximum likelihood estimate of $\dfrac{1}{\theta}$.

(c) Find an unbiased estimate of $\dfrac{1}{\theta}$.

(d) Provide the CRLB for the variances of unbiased estimators of $\dfrac{1}{\theta}$.

(e) Is the estimator you provided in part (c), for $\dfrac{1}{\theta}$, UMVUE? Show that it is or isn't.