In-Class Statistics Masters and Ph.D. Qualifying Exam

January, 2016

Instructions: *The exam has 8 multi-part problems. All of the problems will be graded. Write your ID number on each of your answer sheets. Do not put your name on any of the sheets. Be clear, concise, and complete. All solutions should be rigorously explained.*

Problem 1. (10 pts) In a raffle with 90 tickets, 9 people buy 10 tickets each. If there are 5 winning tickets drawn at random, find the probability that

(a) one person gets all 5 winning tickets.

(b) there are 5 different winners.

Problem 2. (15 pts) X and Y are independent exponential random variables with mean 1. If $V = X + Y$, find the conditional density of X given $V = 3$.

Problem 3. (15 pts) A coin has probability p of coming up heads and $q = 1 - p$ of coming up tails, where $0 < p < 1$. The coin is tossed once and the outcome observed. The coin is then tossed until this outcome is repeated for the first time. Let X denote the total number of tosses that occur (including the first). Find the moment generating function of X.

Problem 4. (10 pts) $X_1, X_2, \ldots$ are iid exponential with mean $\frac{1}{2}$. What is the approximate probability that $P\left(\sum_{i=1}^{100} X_i^2 > 60\right)$?

Problem 5. (10 pts) Suppose $X_1, X_2, \ldots X_n$ are independent and identically distributed as $N(\mu, 1)$. Unfortunately, the observed values $x_1, \cdots, x_n$ were lost; we only have observed values of the random variables

$$Y_i = \begin{cases} 1 & \text{if } X_i \leq 0 \\ 0 & \text{if } X_i > 0 \end{cases}$$

for $i = 1, \cdots, n$. Using only $\{Y_1, \cdots, Y_n\}$, find the MLE of μ.

Problem 6. (10 pts) Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed with continuous pdf $f(x;\theta)$ in the location family; that is, $f(x;\theta) = f_0(x-\theta)$ for all θ where $f_0(x)$ is a pdf.

(a) Show that $X_i - \theta$ is a pivotal quantity for θ.

(b) Show that $\bar{X} - \theta$ and $X_{1:n} - \theta$ are both pivotal quantities for θ.

Problem 7. (15 pts) Suppose $X_1, X_2, \ldots, X_k$ are iid Poisson with mean λ_1; $Y_1, Y_2, \ldots, Y_n$ are iid Poisson with mean λ_2; and all X_i's and Y_j's are independent. Find the size α likelihood ratio test for $H_0 : \lambda_1 = \lambda_2$ vs. $H_A : \lambda_1 \neq \lambda_2$.

Problem 8. (15 pts) We say that $Y \sim \text{LOGN}(\mu, \sigma^2)$ (lognormal) if and only if $X = \ln Y \sim N(\mu, \sigma^2)$. Suppose $Y_1, Y_2, \ldots, Y_n$ with are independent and $Y_i \sim \text{LOGN}(\mu, \sigma^2)$.

(a) What is the distribution of $\prod_{i=1}^{n} Y_i$? Of $\dfrac{Y_1}{Y_2}$?

(b) Show that $E\left[\prod_{i=1}^{n} Y_i\right] = \exp\left[n\left(\mu + \dfrac{\sigma^2}{2}\right)\right]$.

(c) Find the UMVUE of $\theta = \ln(E[Y_1])$ and show that it is the UMVUE.